TRIGONOMETRY FORMULA SHEET

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \cot^2\theta = \csc^2\theta$$
$$\tan^2\theta + 1 = \sec^2\theta$$

Sum/Difference Formulæ

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Double Angle Forulæ

⇒ These can be derived from the sum/diff formulas.

$$\begin{aligned} \sin 2\theta &= 2\sin\theta\cos\theta \\ &= \frac{2\tan\theta}{1+\tan^2\theta} \end{aligned}$$

$$\begin{aligned} \cos 2\theta &= cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta \\ &= \frac{1-\tan^2\theta}{1+\tan^2\theta} \end{aligned}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\cot 2\theta = \frac{\cot\theta - \tan\theta}{2}$$

Power Reduction Formulæ

⇒ These are obtained from the cosine double angle formulas.

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$
$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

Half Angle Formulæ

$$\sin\left(\tfrac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\left(\tfrac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\begin{aligned}\tan\left(\frac{\theta}{2}\right) &= \csc\theta - \cot\theta \\ &= \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \\ &= \frac{\sin\theta}{1+\cos\theta} \\ &= \frac{1-\cos\theta}{\sin\theta}\end{aligned}$$

$$\cot\left(\tfrac{\theta}{2}\right) = \csc\theta + \cot\theta$$

Product to Sum

$$\cos\alpha\cos\beta = \frac{\cos(\alpha-\beta)+\cos(\alpha+\beta)}{2}$$

$$\sin\alpha\sin\beta = \frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2}$$

$$\sin\alpha\cos\beta = \frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{2}$$

$$\cos\alpha\sin\beta = \frac{\sin(\alpha+\beta)-\sin(\alpha-\beta)}{2}$$

Sum to Product

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Cofunction Formulæ

$$\sin\left(\tfrac{\pi}{2}-\theta\right) = \cos\theta$$

$$\cos\left(\tfrac{\pi}{2}-\theta\right) = \sin\theta$$

$$\tan\left(\tfrac{\pi}{2}-\theta\right) = \cot\theta$$

$$\csc\left(\tfrac{\pi}{2}-\theta\right) = \sec\theta$$

$$\sec\left(\tfrac{\pi}{2}-\theta\right) = \csc\theta$$

$$\cot\left(\tfrac{\pi}{2}-\theta\right) = \tan\theta$$

Formulas for Negatives

$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$
$\tan(-\theta) = -\tan\theta$
$\csc(-\theta) = -\csc\theta$
$\sec(-\theta) = \sec\theta$
$\cot(-\theta) = -\cot\theta$

$\pi/2$ Phase Shift

$\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$
$\cos\left(\theta + \frac{\pi}{2}\right) = -\sin\theta$
$\tan\left(\theta + \frac{\pi}{2}\right) = -\cot\theta$
$\csc\left(\theta + \frac{\pi}{2}\right) = \sec\theta$
$\sec\left(\theta + \frac{\pi}{2}\right) = -\csc\theta$
$\cot\left(\theta + \frac{\pi}{2}\right) = -\tan\theta$

π Phase Shift

$\sin(\theta + \pi) = -\sin\theta$
$\cos(\theta + \pi) = -\cos\theta$
$\tan(\theta + \pi) = \tan\theta$
$\csc(\theta + \pi) = -\csc\theta$
$\sec(\theta + \pi) = -\sec\theta$
$\cot(\theta + \pi) = \cot\theta$

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$
$b^2 = a^2 + c^2 - 2ac\cos B$
$c^2 = a^2 + b^2 - 2ab\cos C$