Exercise 2.1

Question 1:

Find the principal value of $sin^{-1}\left(-\frac{1}{2}\right)$

Answer 1:

Let $sin^{-1}\left(-\frac{1}{2}\right) = y$, then $sin\, y = -\frac{1}{2} = -sin\left(\frac{\pi}{6}\right) = sin\left(-\frac{\pi}{6}\right)$

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

Therefore, the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is $-\frac{\pi}{6}$.

Question 2:

Find the principal value of $cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Answer 2:

Let $cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = y$, then $\cos y = \frac{\sqrt{3}}{2} = cos\left(\frac{\pi}{6}\right)$

We know that the range of the principal value branch of $\cos^{-1}$ is $[0, \pi]$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

Therefore, the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$.

Question 3:

Find the principal value of $\text{cosec}^{-1}(2)$.

Answer 3:

Let $\text{cosec}^{-1}(2) = y$. then, $\text{cosec}\, y = 2 = \text{cosec}\left(\frac{\pi}{6}\right)$

We know that the range of the principal value branch of cosec^{-1} is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ and $\text{cosec}\left(\frac{\pi}{6}\right) = 2$.

Therefore, the principal value of $\text{cosec}^{-1}(2)$ is $\frac{\pi}{6}$.

Question 4:

Find the principal value of $tan^{-1}\left(-\sqrt{3}\right)$.

Answer 4:

Let $\tan^{-1}\left(-\sqrt{3}\right) = y$, then $\tan y = -\sqrt{3} = -\tan\frac{\pi}{3} = \tan\left(-\frac{\pi}{3}\right)$

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{3}\right) = -\sqrt{3}$

Therefore, the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$ is $-\frac{\pi}{3}$.

Question 5:

Find the principal value of $cos^{-1}\left(-\frac{1}{2}\right)$.

Answer 5:

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y$, then $\cos y = -\frac{1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi-\frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$

We know that the range of the principal value branch of $\cos^{-1}$ is $[0, \pi]$ and $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

Question 6:

Find the principal value of $\tan^{-1}(-1)$.

Answer 6:

Let $\tan^{-1}(-1) = y$. Then, $\tan y = -1 = -\tan\left(\frac{\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right)$

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{4}\right) = -1$

Therefore, the principal value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}$.

Question 7:

Find the principal value of $sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$.

Answer 7:

Let $sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$, then $\sec y = \frac{2}{\sqrt{3}} = sec\left(\frac{\pi}{6}\right)$

We know that the range of the principal value branch of $\sec^{-1}$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ and $sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$.

Therefore, the principal value of $sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.

Question 8:

Find the principal value of $cot^{-1}\sqrt{3}$.

Answer 8:

Let $\cot^{-1}\sqrt{3} = y$, then $\cot y = \sqrt{3} = \cot\left(\frac{\pi}{6}\right)$.

We know that the range of the principal value branch of $\cot^{-1}$ is $(0, \pi)$ and $\cot\left(\frac{\pi}{6}\right) = \sqrt{3}$.

Therefore, the principal value of $\cot^{-1}\sqrt{3}$ is $\frac{\pi}{6}$.

Question 9:

Find the principal value of $cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

Answer 9:

Let $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y$, then $\cos y = -\frac{1}{\sqrt{2}} = -\cos\left(\frac{\pi}{4}\right) = \cos\left(\pi-\frac{\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right)$.

We know that the range of the principal value branch of $\cos^{-1}$ is $[0, \pi]$ and $\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$.

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

Question 10:

Find the principal value of $cosec^{-1}(-\sqrt{2})$.

Answer 10:

Let $cosec^{-1}(-\sqrt{2}) = y$, then $cosec\, y = -\sqrt{2} = -cosec\left(\frac{\pi}{4}\right) = cosec\left(-\frac{\pi}{4}\right)$

We know that the range of the principal value branch of cosec^{-1} is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$-{0} and $\text{cosec}\left(-\frac{\pi}{4}\right) = -\sqrt{2}$.

Therefore, the principal value of $\text{cosec}^{-1}(-\sqrt{2})$ is $-\frac{\pi}{4}$.

Question 11:

Find the value of $tan^{-1}(1) + cos^{-1}\left(-\frac{1}{2}\right) + sin^{-1}\left(-\frac{1}{2}\right)$.

Answer 11:

Let $tan^{-1}(1) = x$, then $\tan x = 1 = tan\frac{\pi}{4}$

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$\therefore tan^{-1}(1) = \frac{\pi}{4}$

Let $cos^{-1}\left(-\frac{1}{2}\right) = y$, then

$$\cos y = -\frac{1}{2} = -cos\frac{\pi}{3} = cos\left(\pi - \frac{\pi}{3}\right) = cos\left(\frac{2\pi}{3}\right)$$

We know that the range of the principal value branch of $\cos^{-1}$ is $[0, \pi]$.

$\therefore cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

Let $sin^{-1}\left(-\frac{1}{2}\right) = z$, then

$$\sin z = -\frac{1}{2} = -sin\frac{\pi}{6} = sin\left(-\frac{\pi}{6}\right)$$

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

Now,

$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6} = \frac{3\pi + 8\pi - 2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$

Question 12:

Find the value of $cos^{-1}\left(\frac{1}{2}\right) + 2sin^{-1}\left(\frac{1}{2}\right)$

Answer 12:

Let $\cos^{-1}\left(\frac{1}{2}\right) = x$, then

$$\cos x = \frac{1}{2} = \cos\frac{\pi}{3}$$

We know that the range of the principal value branch of $\cos^{-1}$ is $[0, \pi]$.

$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Let $\sin^{-1}\left(-\frac{1}{2}\right) = y$, then

$$\sin y = \frac{1}{2} = \sin\frac{\pi}{6}$$

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\therefore sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

Now,

$$cos^{-1}\left(\frac{1}{2}\right) + 2sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + 2 \times \frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}.$$

Question 13:

If $\sin^{-1} x = y$, then

(A) $0 \leq y \leq \pi$ (B) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

(C) $0 < y < \pi$ (D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Answer 13:

It is given that $\sin^{-1} x = y$.

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Therefore, $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.

Hence, the option (B) is correct.

Question 14:

$tan^{-1}\sqrt{3} - sec^{-1}(-2)$ is equal to

(A) π (B) $-\frac{\pi}{3}$

(C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

Answer 14:

Let $tan^{-1}\sqrt{3} = x$, then

$$\tan x = \sqrt{3} = tan\frac{\pi}{3}$$

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$\therefore \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

Let $\sec^{-1}(-2) = y$, then

$$\sec y = -2 = -\sec\frac{\pi}{3} = \sec\left(\pi - \frac{\pi}{3}\right) = \sec\left(\frac{2\pi}{3}\right)$$

We know that the range of the principal value branch of $\sec^{-1}$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

$\therefore \sec^{-1}(-2) = \frac{2\pi}{3}$

Now,

$$\tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$$

Hence, the option (B) is correct.