Frictional Electricity :— The electricity developed in the bodies when they are rubbed with each other is called frictional electricity.

Frictional electricity is also known as static electricity because the charge acquired by the body does not flow from one part of the body to the other part.

Kinds of Electric Charge :—

There are two kinds of charges— positive charge and negative charge.

when two bodies rubbed with each other, they acquired two different kinds of charges i.e. one body acquires positive charge and other body acquires negative charge.

For example, when a glass rod is rubbed with silk, then glass rod acquires positive charge and silk acquires negative charge.

In the following table, when the bodies listed in column I are rubbed with bodies of column II, then bodies of column I acquire positive charge and the bodies of column II acquire negative charge.

Column I (+ve)	Column II (-ve)
Glass Rod	Silk cloth
Fur	Ebonite Rod
Dry Hair	Comb
Wool	Amber
Wool	Rubber Shoes
Wool	Plastic objects

The bodies having same kind of charge repel each other and the bodies having opposite kind of charge attract each other.

Quantization of Charge :— The charge on a body is always equal to integral multiple of a smallest unit of charge. The smallest unit of charge is equal to the charge on an electron or proton.

The charge on electron is $1.6 \times 10^{-19} C$ and is negative
The charge on proton is $1.6 \times 10^{-19} C$ and is positive.

Thus, a body always possesses charge,

$$q = \pm ne$$

where $n = 1, 2, 3$ ———

and $e = 1.6 \times 10^{-19} C$ [e = elementary charge]

This experimental fact is called quantization of charge.

Thus, the charge on body exists in the form of discrete packets of charge and is not continuous.

Electric charge in additive :— Positive and negative charges are added like real numbers i.e. total charge on a body is equal to the algebric sum of all the charges present anywhere in the body. This shows the additive nature of charges.

Thus two charges $+3q$ and $+q$ on a body add up to $+4q$.

The two charges $+3q$ and $-q$ on a body add up to $+2q$.

Invariance of Electric charge :—

Electric charge is independent of frame of reference i.e. the electric charge on a particle is not affected by the motion of the particle.

Conservation Of Charge :— Law of conservation of charge states that, " The algebric sum of electric charges on all the bodies in an isolated system always remain constant."

In other words," Charge can neither be created nor be destroyed in an isolated system." Examples :—

① When a glass rod is rubbed with a piece of silk cloth, glass rod becomes positively charged and silk cloth becomes negati

-vely charged. Before rubbing, both glass rod and silk cloth were neutral i.e. total charge on the system (glass rod + silk cloth) was zero. On rubbing, glass rod transfers some electrons to the silk cloth. It means glass rod after losing electrons becomes +vely charged and silk cloth after receiving electrons becomes negatively charged. The algebric sum of charges on glass rod and silk cloth after rubbing continues to be zero. Hence total charge is conserved.

(2) The case of radioactive decay, the uranium ${}_{92}U^{238}$ decays as:

$$ {}_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4} $$

In this reaction :-

Total charge before decay $= +92e$

Total charge after decay $= +90e + 2e$
$= 92e$

Hence the total charge is conserved.

Properties of Electric Charge :-

(1) There are two kinds of charges +ve and -ve.

(2) Like charges repel each other, while unlike charges attract each other.

(3) The electric charge is additive in nature i.e. the total charge on a body is equal to the algebraic sum of all the charges present anywhere in the body.

(4) The charge is quantized i.e. the charge on a body is always equal to integral multiple of 'e' i.e.

$$ q = \pm ne, \text{ where } n = 1, 2, 3, 4, \ldots $$

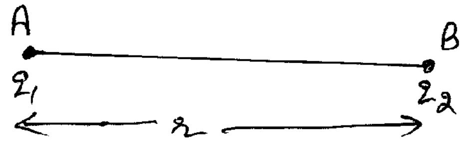
(5) In an isolated system, charge is always conserved.

(6) The electric charge on a particle is not affected by the motion of the particle.

Coulomb's Law of Electrostatic Force :—

According to coulomb's law of electrostatic force, the force of attraction or repulsion between any point charges at rest is directly proportional to the product of the magnitude of charges and inversely proportional to the square of distance between them.

If two point charges q_1 and q_2 be situated at points A and B respectively and let r be the distance between them.



Then, Force between charges is given by :—

$$F \propto q_1 q_2 \quad \text{——— (1)}$$

$$F \propto \frac{1}{r^2} \quad \text{——— (2)}$$

Combining (1) and (2)

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\text{or} \quad F = K \frac{q_1 q_2}{r^2} \quad \text{——— (3)}$$

where K is a constant of proportionality and is known as electrostatic force constant.

The value of K depends upon the system of units in which other quantities F, r, q_1 and q_2 are measured. Its value also depend upon the medium in which two charges are placed.

In S.I system :— when two charges lying in vaccum (or air) then value of $K = \frac{1}{4\pi\epsilon_0}$, where ϵ_0 (Epsilon-zero) is called absolute permitivity of free space.

Putting this value in (3) we get

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \quad \text{——— (4)}$$

The value of $\epsilon_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$ and the value of $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9\ Nm^2 C^{-2}$

∴ Equation (4) for air or vacum can be written as

$$F = 9\times10^9 \frac{q_1 q_2}{r^2} \text{ Newton} \longleftarrow (5)$$

In C.G.S System :—

$$k = \frac{1}{K}$$

where K = dielectric constant of the medium.

For air or vacuum, k = 1

∴ when two point charges are placed in vacuum or air, then equation (3) in c.g.s system becomes

$$F = \frac{q_1 q_2}{r^2} \quad [\because k = 1 \text{ for air or vacuum}]$$

Units of Charge :—

(1) S.I unit of charge is coulomb.

The force between two charges q_1 and q_2 when placed in air or vacuum at a distance r, is given by

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

If $q_1 = q$ and $q_2 = q$ (say), $r = 1\,m$

$F = 9\times10^9 N$, $\frac{1}{4\pi\epsilon_0} = 9\times10^9\ Nm^2C^{-2}$

Then above equation becomes :—

$$9\times10^9 N = (9\times10^9\ Nm^2C^{-2}) \times \frac{q\times q}{(1)^2 m^2}$$

or $q^2 = 1\,C^2$

$$\Rightarrow \boxed{q = \pm 1C}$$

Thus, one coulomb is that charge which repels equal and similar charge placed at a distance of 1m from it in vacuum or air with a force of $9\times10^9 N$.

(2) C.g.s unit of charge is stat-coulomb or electrostatic unit (e.s.u) of charge.

The force between two charges q_1 and q_2 when placed in air or vacuum at a distance r, is given by

$$F = \frac{q_1 q_2}{r^2}$$

If $q_1 = q$ and $q_2 = q$, $r = 1$ cm, $F = 1$ dyne

Then above equation becomes

$$1 = \frac{q \times q}{(1)^2}$$

$$\Rightarrow q^2 = 1$$

$$\Rightarrow \boxed{q = \pm 1 \text{ stat-coulomb}}$$

Thus one stat-coulomb or one e.s.u of charge is that charge which repels equal and similar charge placed at a distance of 1 cm from it in vacuum or air with a force of 1 dyne.

Relation 1 Coulomb $= 3 \times 10^9$ stat-coulomb

1 Coulomb $= 3 \times 10^9$ e.s.u of charge.

(3) Electro-magnetic unit of charge (e.m.u) :-

The unit is electro magnetic unit of charge (e.m.u)

1 e.m.u of charge $= 10$ Coulomb

$= 10 \times 3 \times 10^9$ stat-coulomb

$= 3 \times 10^{10}$ stat-coulomb

Dielectric Constant or Relative Permittivity :-

When two charges are placed in a medium other than air or vacuum, then the force between two charges may change.

If two point charges q_1 and q_2 are placed at a distance r in a medium, then force between them is given by

$$F_{med} = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2} \quad \text{——— (1)}$$

where ϵ = Permittivity of medium.

Force between two same charges at the same distance r in vacuum is given by

$$F_{vac} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \quad \text{——— (2)}$$

Equation (2) by (1), we get

$$\frac{F_{vac}}{F_{med}} = \frac{\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}}{\frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2}} = \frac{\epsilon}{\epsilon_0}$$

$$\frac{F_{vac}}{F_{med}} = \frac{\epsilon}{\epsilon_0} = \epsilon_r \,(= k) \quad \text{—— (3)}$$

The ratio $\frac{\epsilon}{\epsilon_0} = \epsilon_r$ (or k) is called relative permittivity of the medium w.r.t vacuum. It is also called dielectric constant (k) of the medium.

From (3),

$$\frac{F_{vac}}{F_{med}} = \epsilon_r \,(\text{or } k)$$

The dielectric constant (k) or relative permittivity (ϵ_r) of the medium may be defined as the ratio of the Coulomb's force between two point charges placed in air or vacuum to the Coulomb's force between the same charges placed in the medium separated by same distance from each other.

From relation, $\frac{\epsilon}{\epsilon_0} = \epsilon_r$

we also defined the relative permittivity of the medium as the ratio of permittivity of medium to the absolute permittivity of free space (air or vacuum).

ϵ_r or k has no unit. Its value depend upon the nature of medium.

For vacuum, $k = 1$ and for air, $k = 1.0006$

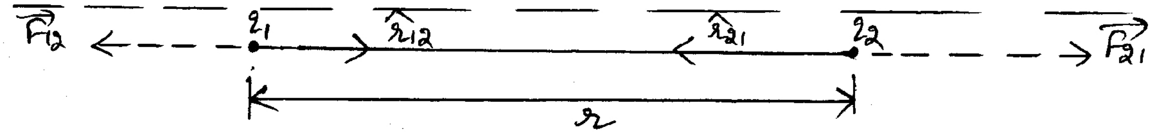
for glass, $k = 5$ to 8

and for water, $k = 80$

Coulomb's law in vector Form :—

(Coulomb's law in accordance with Newton's Third law) :—

Consider two point charges q_1 and q_2 separated by a distance r.



Let $\vec{F_{12}}$ = Force acting on charge q_1 due to q_2.

$\vec{F_{21}}$ = Force acting on charge q_2 due to q_1.

$\hat{r}_{12}$ = unit vector from q_1 to q_2.

$\hat{r}_{21}$ = unit vector from q_2 to q_1.

Now $\vec{F_{21}}$ and $\hat{r}_{12}$ are in same direction

$$\therefore \vec{F_{21}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12} \quad \text{--- (1)}$$

Also, $\vec{F_{12}}$ and $\hat{r}_{21}$ are in same direction

$$\therefore \vec{F_{12}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21} \quad \text{--- (2)}$$

The equations (1) and (2) gives the Coulomb's law in vector form.

From (1), magnitude of force is

$$|\vec{F_{21}}| = \left| \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12} \right|$$

$$\Rightarrow F_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \quad \text{--- (3)}$$

From (2), magnitude of force is

$$|\vec{F_{12}}| = \left| \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21} \right|$$

$$\Rightarrow F_{12} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \quad \text{--- (4)}$$

From (3) & (4), we get

$$F_{21} = F_{12} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

Since $\hat{r}_{12}$ and $\hat{r}_{21}$ are unit vectors, opposite to each other.

i.e. $\hat{r}_{21} = -\hat{r}_{12}$

Put this value of $\hat{r}_{21}$ in (2), we get

$$\vec{F_{12}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} (-\hat{r}_{12})$$

$$= -\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

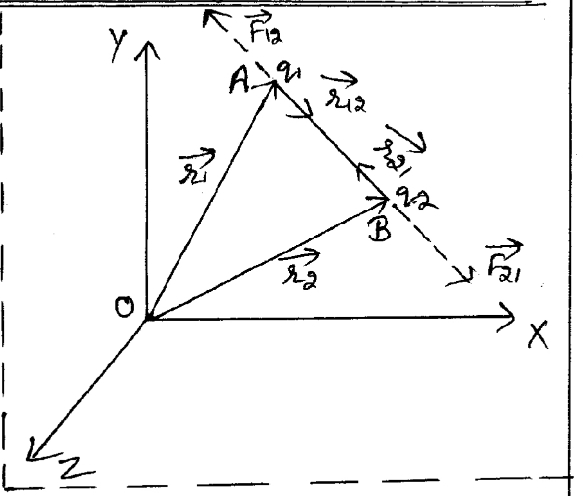
$\Rightarrow \vec{F_{12}} = -\vec{F_{21}}$, Thus the forces exerted by two charges on each other are equal and opposite. Therefore Newton's third law of motion is obeyed.

Forces between two point charges in terms of their position vector :—

Let two point charges q_1 and q_2 are located at A and B in space.

let $\vec{OA} = \vec{r_1}$ = Position vector of charge q_1 w.r.t origin O.

and $\vec{OB} = \vec{r_2}$ = Position vector of charge q_2 w.r.t origin O.



According to triangle law of vectors,

$$\vec{OA} + \vec{AB} = \vec{OB}$$

or $$\vec{AB} = \vec{OB} - \vec{OA}$$

or $$\vec{AB} = \vec{r_2} - \vec{r_1} = \vec{r_{12}} \quad \text{—— (1)}$$

According to Coulomb's law, force on q_2 due to q_1 is

$$\vec{F_{21}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{AB}|^2} \hat{r}_{12}$$

or $$\vec{F_{21}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r_{12}}|^2} \hat{r}_{12} \quad \text{—— (2)}$$

or $$\vec{F_{21}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r_{12}}|^2} \times \frac{\vec{r_{12}}}{|\vec{r_{12}}|} \quad \left[\because \hat{a} = \frac{\vec{a}}{|\vec{a}|} \right]$$

or $$\vec{F_{21}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|\vec{r_{12}}|^3} \cdot \vec{r_{12}} \quad \text{—— (3)}$$

Similarly, $$\vec{F_{12}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|\vec{r_{21}}|^3} \vec{r_{21}} \quad \text{—— (4)}$$

Using $\vec{r_{12}} = \vec{r_2} - \vec{r_1}$ from (1), equation (3) can be written as :-

$$\vec{F_{21}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r_2} - \vec{r_1}|^3} (\vec{r_2} - \vec{r_1}) \quad \text{—— (5)}$$

Similarly, equation (4) can be written as

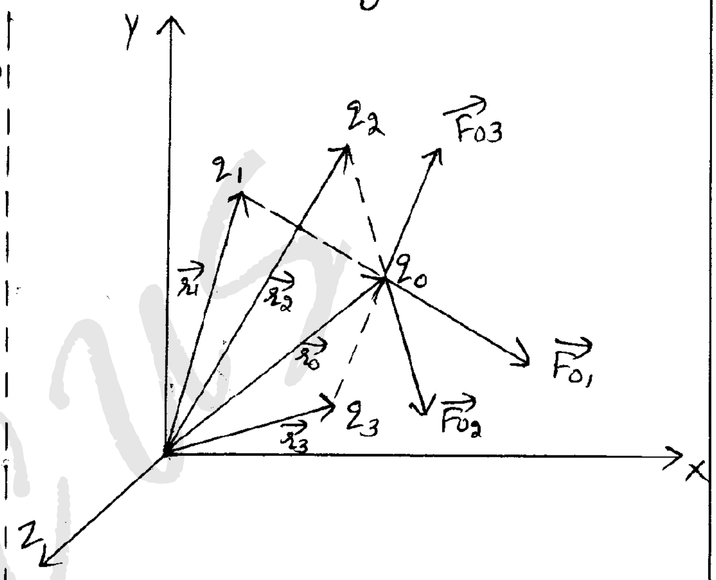
$$\vec{F_{12}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^3} (\vec{r_1} - \vec{r_2}) \quad \text{—— (6)}$$

These equations (5) & (6) represent coulomb's law in terms of position vectors.

Principle of Superposition :— The principle of superposition states that, " when a number of point charges are interacting, the total force on a particular charge is the vector sum of forces exerted on it by all other charges."

Let us consider $q_1, q_2, q_3 ---- q_n$ be the n- point charges. placed at points with position vectors $\vec{r_1}, \vec{r_2}, \vec{r_3}, ----, \vec{r_n}$ respectively w.r.t origin 'O'.

q_0 be the particular charge where forces due to all other charges is to be calculated.



Let $\vec{r_0}$ be the position vector of charge q_0.

Force on q_0 due to q_1 is given by

$$\vec{F_{01}} = \frac{1}{4\pi\epsilon_0} \frac{q_0 q_1}{|\vec{r_0} - \vec{r_1}|^3} (\vec{r_0} - \vec{r_1})$$

and force on q_0 due to q_2 is given by

$$\vec{F_{02}} = \frac{1}{4\pi\epsilon_0} \frac{q_0 q_2}{|\vec{r_0} - \vec{r_2}|^3} (\vec{r_0} - \vec{r_2})$$

Similarly force on q_0 due to q_3 is given by

$$\vec{F_{03}} = \frac{1}{4\pi\epsilon_0} \frac{q_0 q_3}{|\vec{r_0} - \vec{r_3}|^3} (\vec{r_0} - \vec{r_3})$$

$\therefore$ Force on q_0 due to q_n is given by

$$\vec{F_{0n}} = \frac{1}{4\pi\epsilon_0} \frac{q_0 q_n}{|\vec{r_0} - \vec{r_n}|^3} (\vec{r_0} - \vec{r_n})$$

$\therefore$ Total force on q_0 due to all other charges is

$$\vec{F_0} = \vec{F_{01}} + \vec{F_{02}} + \vec{F_{03}} + \text{-----} + \vec{F_{0n}}$$

$$\vec{F_0} = \frac{1}{4\pi\epsilon_0}\frac{q_0 q_1}{|\vec{r_0}-\vec{r_1}|^3}(\vec{r_0}-\vec{r_1}) + \frac{1}{4\pi\epsilon_0}\frac{q_0 q_2}{|\vec{r_0}-\vec{r_2}|^3}(\vec{r_0}-\vec{r_2})$$

$$+ \frac{1}{4\pi\epsilon_0}\frac{q_0 q_3}{|\vec{r_0}-\vec{r_3}|^3}(\vec{r_0}-\vec{r_3}) + \text{---} + \frac{1}{4\pi\epsilon_0}\frac{q_0 q_n}{|\vec{r_0}-\vec{r_n}|^3}(\vec{r_0}-\vec{r_n})$$

$$\Rightarrow \vec{F_0} = \frac{q_0}{4\pi\epsilon_0}\sum_{i=1}^{i=n}\frac{q_i(\vec{r_0}-\vec{r_i})}{|\vec{r_0}-\vec{r_i}|^3}$$

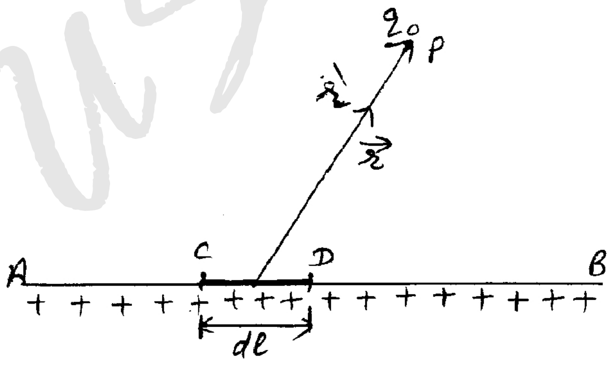
Electrostatic force due to continuous charge distribution :-

There are three types of charge distribution :-

(1) **Linear charge distribution** :- when the charge is distributed along a line (straight or curve), the charge distribution is known as linear distribution.

Consider a straight wire AB of length l on which charge q is distributed uniformly.

Let us take a small element CD of length 'dl' on the wire.



If λ is linear charge density, then charge on element of length dl is given by

$$\lambda = \frac{dq}{dl}$$

$$\Rightarrow dq = \lambda\, dl \quad \text{——} (1)$$

$\left[\because \text{Linear charge density } \lambda = \frac{\text{charge}}{\text{length}}\right]$

Let P be a point at a distance 'r' from element CD, where a test charge q_0 is placed.

Now, the electrostatic force on charge q_0 due to element of charge dq is given by

$$\vec{dF} = \frac{1}{4\pi\epsilon_0}\frac{q_0\, dq}{r^2}\hat{r}$$

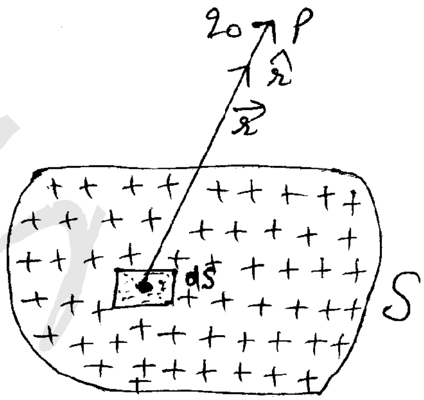
$$\Rightarrow \vec{dF} = \frac{1}{4\pi\epsilon_0}\frac{q_0 \lambda\, dl}{r^2}\hat{r} \quad \left[\because \text{of } (1)\ dq = \lambda dl\right]$$

$\therefore$ Total force on charge q_0 due to the charge on the whole wire is

$$\int d\vec{F} = \int_l \frac{1}{4\pi\epsilon_0} \frac{q_0 \lambda\, dl}{r^2} \hat{r}$$

$$\Rightarrow \vec{F} = \frac{q_0}{4\pi\epsilon_0} \int_l \frac{\lambda\, dl}{r^2} \cdot \hat{r} \quad \text{—— (2)}$$

(2) **Surface charge distribution** :— when the charge is distributed continuously over a surface, then charge distribution is called surface charge density.



Consider a surface of area S on which charge q is distributed uniformly.

Let us take a small element of area 'ds' on the given surface.

If σ = surface charge density, then charge on the element of area ds is given by

$$\sigma = \frac{dq}{ds}$$

$$\Rightarrow dq = \sigma\, ds \quad \text{—— (3)}$$

$\left[\because \text{ surface charge density } \sigma = \frac{\text{charge}}{\text{area}}\right]$

Let P be a point at a distance 'r' from the element of area ds, where a test charge q_0 is placed.

The electrostatic force on charge q_0 due to element of charge dq is given by

$$d\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_0\, dq}{r^2} \hat{r}$$

$$\Rightarrow d\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_0 \sigma\, ds}{r^2} \hat{r} \quad \left[\because \text{ of (3) } dq = \sigma\, ds\right]$$

∴ Total force on charge q_0 due to the charge on the whole surface is

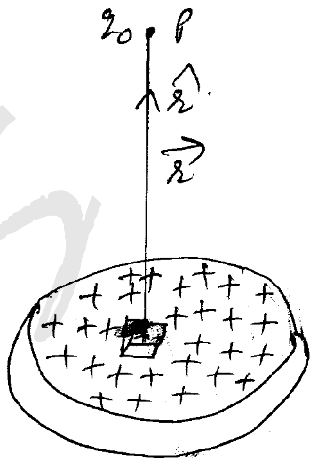
$$\int d\vec{F} = \int_S \frac{1}{4\pi\epsilon_0} \frac{q_0\, \sigma\, dS}{r^2} \hat{r}$$

$$\Rightarrow \vec{F} = \frac{q_0}{4\pi\epsilon_0} \int_S \frac{\sigma\, dS}{r^2} \hat{r} \quad \text{——— (4)}$$

(3) Volume charge distribution :— When the charge is continuously distributed on a volume of an object, then distribution is called volume charge density.

Consider a surface of volume V having charge q which is uniformly distributed over it.

If ρ = volume charge density, then charge on the element of area dV is given by



$$\rho = \frac{dq}{dV}$$

$$\Rightarrow dq = \rho\, dV \quad \text{——— (6)}$$

[∵ volume charge density $\rho = \frac{\text{charge}}{\text{volume}}$]

Let P be a point at a distance 'r' from the element of volume dV, where a test charge q_0 is placed.

The electrostatic force on charge q_0 due to element of charge dq is given by

$$d\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_0\, dq}{r^2} \hat{r}$$

$$\Rightarrow d\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_0\, \rho\, dV}{r^2} \hat{r} \quad \left[\because \text{ of (6) } dq = \rho\, dV \right]$$

∴ Total force on charge q_0 due to the charge on the whole surface is

$$\int d\vec{F} = \int_V \frac{1}{4\pi\epsilon_0} \cdot \frac{q_0 \rho\, dv}{r^2}\, \hat{r}$$

or $$\vec{F} = \frac{q_0}{4\pi\epsilon_0} \int_V \frac{\rho\, dv}{r^2}\, \hat{r}$$