1(b) Electrostatic Field

Concept of Electric field :– The space, around a charged body within which other charge can experience electrostatic force, is called electric field.

OR

The region or space around a charged body within which its influence can be felt is called electric field.

If a test charge experiences no force at a point, the electric field at that point must be zero.

* **Test charge :–** The very small charge experiencing force due to electric field is called test charge.

* **Source Charge :–** The charge producing electric field is known as source charge.

Electric field intensity (Strength of electric field, Electric intensity):–
(Relation between electric field intensity and force)

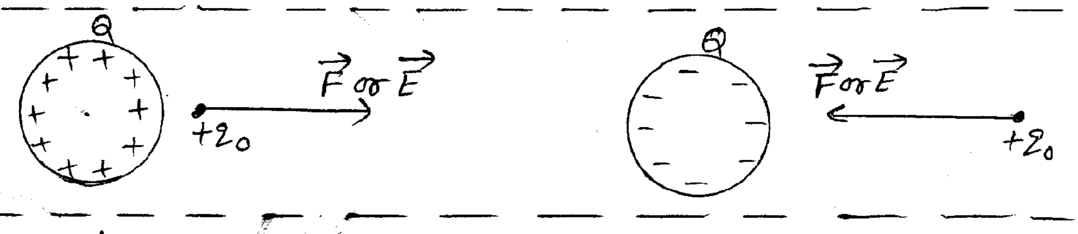
The electric field intensity at any point is defined as the force experienced by a unit positive charge (or a test charge (q_0)) placed at that point.

If $\vec{F}$ is the force acting on a test charge ($+q_0$) at a point, then electric field intensity at a point is given by

$$\vec{E} = \frac{\vec{F}}{q_0} \quad \text{——— (1)}$$

Electric field intensity ($\vec{E}$) is a vector quantity.

The direction of $\vec{E}$ is the same as the direction of $\vec{F}$, i.e. $\vec{E}$ is along the direction in which test charge ($+q_0$) would tend to move.



In equation (1), $q_0 \to 0$ (i.e. q_0 is very-very small), so that presence of this charge may not affect the source charge Q and its electric field is not changed. Therefore, expression for electric field intensity can be written as :–

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} \quad \text{——— (2)}$$

In magnitude, equation (1) can be written as

$$|\vec{E}| = \left|\frac{\vec{F}}{q_0}\right|$$

$$\Rightarrow E = \frac{F}{q_0} \quad \text{——— (3)}$$

Units of electric field intensity :-

In S.I system, unit of electric field intensity is N/C or NC^{-1} (Newton per coulomb)

In c.g.s system, unit of electric field intensity is dyne/Stat-coulomb

Dimensional formula of electric field intensity :-

From (3)

$$E = \frac{F}{q_0} = \frac{\text{Force}}{\text{charge}} = \frac{\text{Force}}{\text{Current} \times \text{time}} \quad \left[\because \text{Current} = \frac{\text{Charge}}{\text{Time}}\right]$$

$$\therefore \text{D.F. of electric field intensity} = \frac{[MLT^{-2}]}{[A][T]}$$

$$= [MLT^{-2}A^{-1}T^{-1}]$$

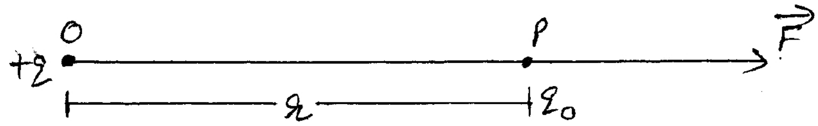
$$= [MLT^{-3}A^{-1}]$$

[A = Dimensional formula of current (ampere)]

Electric field intensity at a point due to a point charge :-

Let $+q$ be a point charge placed at point 'O'. Due to this charge an electric field is developed.

Let P be a point in the field, where electric field intensity is to be determined.



Let $OP = r$

Let a small positive test charge q_0 is placed at P.

Then using coulomb's law, the force at q_0 (at P) due to q (at O) is given by :-

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}\hat{r}$$

$\hat{r}$ is a unit vector directed from O to P.

$\therefore$ Electric field intensity at point P is given by

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q q_0}{q_0 r^2} \hat{r}$$

$$\Rightarrow \boxed{\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}}$$

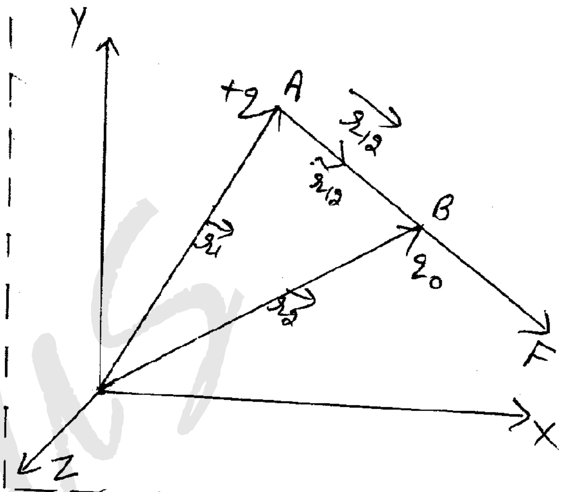
Electrostatic field intensity at a point due to a point charge in terms of position vector :–

Let $+q$ be a point charge placed at a point A.

i.e. $\vec{OA} = \vec{r_1}$

and q_0 be a test charge placed at point B (i.e) $\vec{OB} = \vec{r_2}$

we have to calculate electric field intensity ($\vec{E}$) at point B.



Thus, using Coulomb's law, the force at q_0 (at B) due to $+q$ (at A) is given by

$$\vec{F_{q_0 q}} = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{|\vec{r_{12}}|^2} \hat{r_{12}} \quad \text{——— (1)}$$

$$\text{or } \vec{F_{q_0 q}} = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{|\vec{r_{12}}|^2} \cdot \frac{\vec{r_{12}}}{|\vec{r_{12}}|} \quad \left[\because \hat{a} = \frac{\vec{a}}{|\vec{a}|}\right]$$

$$\text{or } \vec{F_{q_0 q}} = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{|\vec{r_{12}}|^3} \vec{r_{12}} \quad \text{——— (2)}$$

From triangle law of vectors, in ΔOAB,

$$\vec{OA} + \vec{AB} = \vec{OB}$$

$$\Rightarrow \vec{r_1} + \vec{r_{12}} = \vec{r_2}$$

$$\text{or } \vec{r_{12}} = \vec{r_2} - \vec{r_1}$$

Putting this value of $\vec{r_{12}}$ in (2), we get

$$\vec{F_{q_0 q}} = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{|\vec{r_2} - \vec{r_1}|^3} (\vec{r_2} - \vec{r_1})$$

Electric field intensity at q_0 (at B) due to $+q$ (at A) is given by

$$\vec{E_{q_0 q}} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q\, q_0}{q_0 |\vec{r_2} - \vec{r_1}|^3} (\vec{r_2} - \vec{r_1})$$

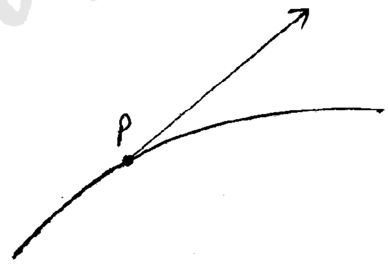
or $$\boxed{\vec{E_{q_0 q}} = \frac{1}{4\pi\epsilon_0} \frac{q}{|\vec{r_2} - \vec{r_1}|^3} (\vec{r_2} - \vec{r_1})}$$

Electric Lines of force :-

An electric line of force is the path along which a unit positive charge would move, if it is free to do so.

The electric line of force points in the direction of electric field. The electric line of force may be straight or curved. In case, electric line of force is a curve, then direction of electric field at any point is given by the tangent to the line of force at that point as shown in figure :-

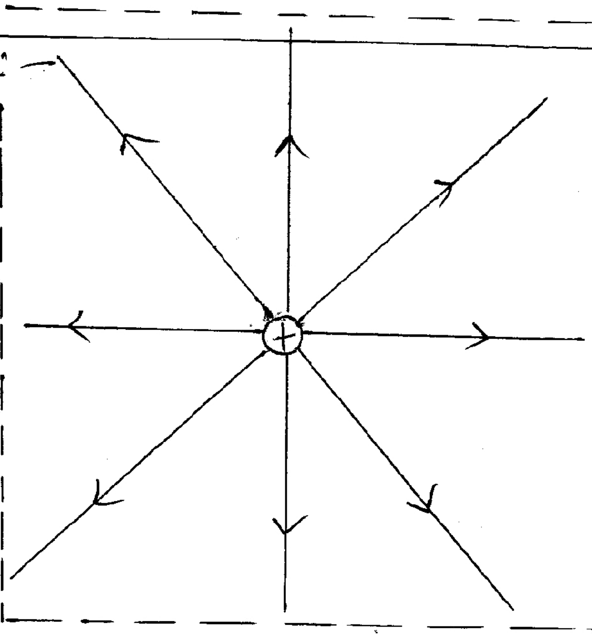
The strength of the electric field at any point is measured as the number of lines of force crossing a unit area held normal to the lines of force at that point.



Lines of force due to single charge :-

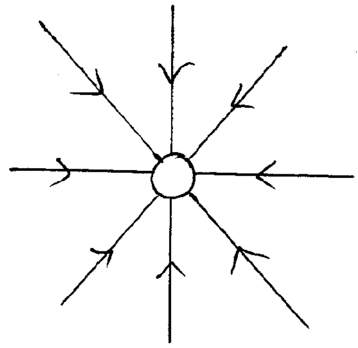
The electric lines of force or field lines due to single positive charge. (Shown in figure)

These lines of force are directed radially outward.



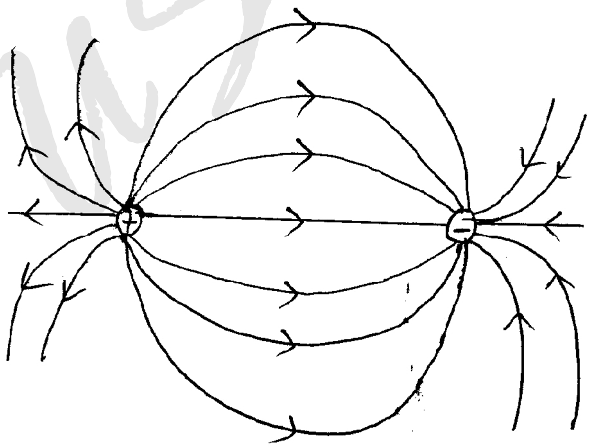
The electric lines of force due to single negative charge.

These lines of force are directed radially inwards.

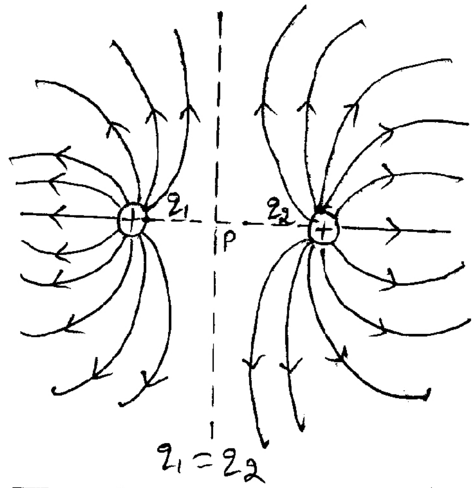


Field lines due to a system of two charges :-

The electric lines of force due to equal and opposite charges +q and −q as shown in figure. The electric lines of force tends to contract lengthwise, which shows that opposite charges attract each other



The lines of force due to two equal positive point charges as shown in figure. The force lines due to the two equal and like charges exert lateral pressure on each other, which is responsible for repulsion between two like charges



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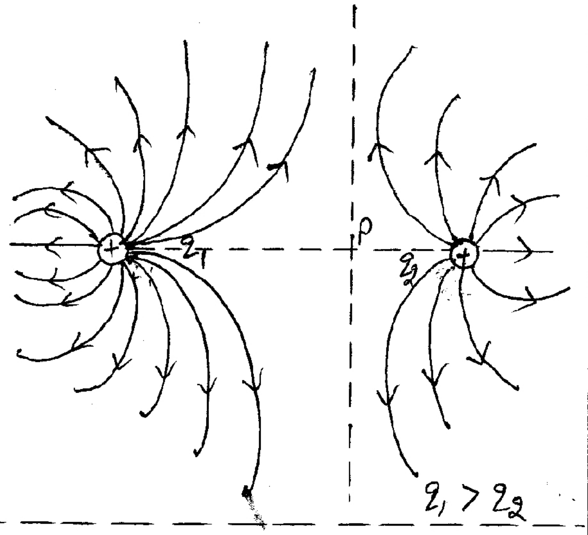
The lines of force due to two unequal positive point charges are shown. The force lines due to two unequal and like charges exert lateral pressure on each other, which is reponsible for repulsion between two like charges. In this case, when charges are unequal, the neutral point is closer to the small charge.



Properties of electric lines of force or field lines :—

(i) The lines of force start from the positive charge and end at the negative charge.

(ii) The lines of force originate or terminate at a surface always at a surface always at right angles to the surface.

(iii) The tangent to the line of force at any point gives the direction of the electric field at that point.

(iv) Two electric lines of force can not intersect (cross) each other. If two electric lines of force cross each other, then at the point of intersection P, there will be two tangents, which means that there are two values of the electric field at that point, which is not possible. see in fig. (a)

(v) whenever the electric lines of force are

(a) Closer, the electric field is strong.

(b) For apart, the electric field is weak

(c) Parallel and equally spaced, the electric field is uniform.

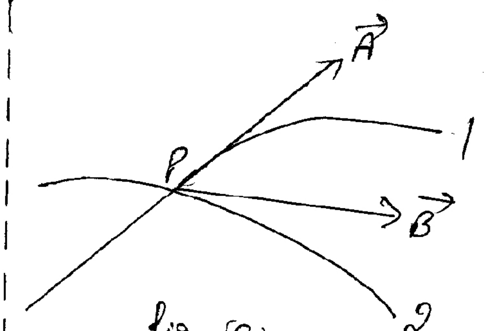


fig. (a)

(vi) The lines of force do not pass through a conductor. It shows that the electric field inside a conductor is always zero.

(vii) The lines of force contract lengthwise. This property of lines of force leads to explain attraction between two unlike charges.

(viii) The lines of force exert a lateral pressure on each other. This property of lines of force leads to explain the repulsion between two like charges.

(ix) Electric lines of force are imaginary and do not exist in reality. But the electric field they represent is real.

(x) The electric field intensity at any point in an electric field may be defined as the number of lines of force crossing a unit area held normal to the lines of force at that point.

Importance of electric lines of force :- The electric lines of force can be used to represent the electric field due to a point charge or a system of point charges.

Electric dipole :- A system of two equal and opposite charges separated by a fixed distance is called an electric dipole.

Example :- The molecules of water, ammonia, HCl etc, behaves as electric dipole. It is because, the centres of positive and negative charges in these molecules lie at a small distance from each other.

The figure shows an electric dipole having charges $-q$ and $+q$ separated by a small distance '$2a$'.

The distance '$2a$' is called length of the dipole or dipole length.

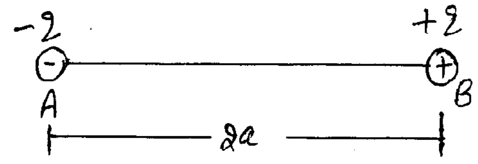


fig. (b)

Electric Dipole moment :- Electric dipole moment is defined as the product of magnitude of either charge and the length of the electric dipole.

The above figure (fig (b)) shows an electric dipole having charges $-q$ and $+q$ separated by a small distance '$2a$'.

Then electric dipole moment ($\vec{p}$) is given by

$$\vec{p} = q\,(2\vec{a})$$

Electric dipole moment is a vector quantity and its direction is from the negative to the positive charge.

In S.I system, unit of electric dipole moment is coulomb metre (cm)

In c.g.s system, unit of electric dipole moment is stat-coulomb centimetre.

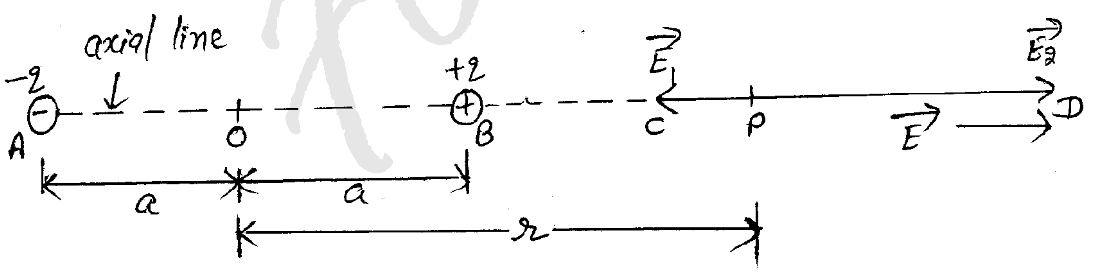
The electric field produced by a dipole is called dipole field.

Electric dipole moment ($\vec{p}$) is a measure of the strength of the electric dipole.

Electric field or electric field intensity on axial line of an electric dipole :-

A line passing through the positive and negative charges of the electric dipole is called the axial line of the electric dipole.

Consider an electric dipole having charges $-q$ and $+q$ separated by a distance '$2a$'. Let P be a point on the axial line of the dipole at a distance 'r' from the centre O of the dipole. Let a unit positive charge be placed at point P.



Let $OP = r$

$\because AB = 2a \qquad \therefore AO = OB = a$

$\therefore BP = (r-a), \quad AP = (r+a)$

Electric field intensity at P due to charge '$-q$' is given by

$$\vec{E_1} = \frac{1}{4\pi\epsilon_0}\frac{q}{(AP)^2} \text{ along } \vec{PC}$$

or $\vec{E_1} = \frac{1}{4\pi\epsilon_0} \frac{q}{(r+a)^2}$ along $\vec{PC}$

or $|\vec{E_1}| = \left| \frac{1}{4\pi\epsilon_0} \frac{q}{(r+a)^2} \text{ along } \vec{PC} \right|$

or $E_1 = \frac{1}{4\pi\epsilon_0} \frac{q}{(r+a)^2}$ —— (1)

Electric field intensity at P due to charge '+q' is given by

$$\vec{E_2} = \frac{1}{4\pi\epsilon_0} \frac{q}{(BP)^2} \text{ along } \vec{PD}$$

or $\vec{E_2} = \frac{1}{4\pi\epsilon_0} \frac{q}{(r-a)^2}$ along $\vec{PD}$

or $|\vec{E_2}| = \left| \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r-a)^2} \text{ along } \vec{PD} \right|$

or $E_2 = \frac{1}{4\pi\epsilon_0} \frac{q}{(r-a)^2}$ —— (2)

It is clear that $E_2 > E_1$ because point P lies near to B.

Resultant electric field intensity at P due to dipole is

$$E = E_2 - E_1$$

or $E = \frac{1}{4\pi\epsilon_0} \frac{q}{(r-a)^2} - \frac{1}{4\pi\epsilon_0} \frac{q}{(r+a)^2}$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} \right]$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{(r+a)^2 - (r-a)^2}{(r-a)^2 (r+a)^2} \right]$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{r^2+a^2+2ra-(r^2+a^2-2ra)}{(r^2-a^2)^2} \right]$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{r^2+a^2+2ra-r^2-a^2+2ra}{(r^2-a^2)^2} \right]$$

or $$E = \frac{q}{4\pi\epsilon_0} \frac{4ra}{(r^2 - a^2)^2}$$

or $$E = \frac{1}{4\pi\epsilon_0} \frac{(q \cdot 2a)\, 2r}{(r^2 - a^2)^2}$$

or $$E = \frac{1}{4\pi\epsilon_0} \frac{p \cdot 2r}{(r^2 - a^2)^2}$$ [In magnitude $\because p = q \cdot 2a$]

or In vector Form :-

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot 2r}{(r^2 - a^2)^2} \text{ along } \overrightarrow{PD} \quad \text{—— (3)}$$

The direction of electric field intensity at a point on the axial line is from $-q$ to $+q$ charge i.e. same as the direction of electric dipole moment.

If dipole is very small i.e. $a << r$, then in equation (3), a^2 can be neglected as compared to r^2

$$\therefore \vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\vec{p} \cdot 2r}{(r^2)^2} \text{ along } \overrightarrow{PD}$$

or $$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\vec{p} \cdot 2r}{r^4} \text{ along } \overrightarrow{PD}$$

or $$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2\vec{p}}{r^3} \text{ along } \overrightarrow{PD}$$

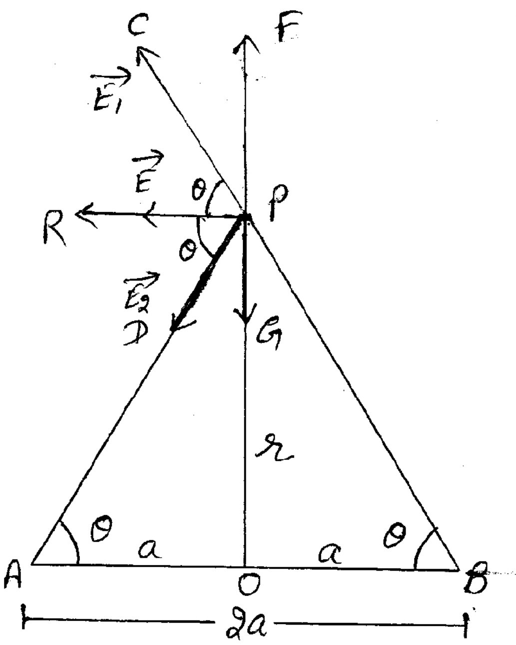
In magnitude, $$E = \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3}$$

Electric field on equitorial line of an electric dipole :-

An equitorial line of the electric dipole is a line perpendicular to the axial line and passes through the centre of the electric dipole.

Consider an electric dipole AB, having two charges $-q$ and $+q$ separated by a small distance '$2a$'.

Let O be the mid-point of dipole AB. P be a point on the equilateral line of dipole, where electric intensity is to be determined.



Let $OP = r$, $AO = BO = a$

$\therefore AP = BP = \sqrt{r^2 + a^2}$

$$\boxed{\begin{aligned} &\because BP^2 = BO^2 + PO^2 \\ &BP^2 = a^2 + r^2 \\ &\Rightarrow BP = \sqrt{a^2 + r^2} \\ &\Rightarrow BP = \sqrt{r^2 + a^2} \end{aligned}}$$

Let a unit positive charge be placed at P.

Electric field intensity at P due to charge '+q' is

$$\vec{E_1} = \frac{1}{4\pi\epsilon_0} \frac{q}{(BP)^2} \text{ along PC}$$

$$\text{or } \vec{E_1} = \frac{1}{4\pi\epsilon_0} \frac{q}{(\sqrt{r^2+a^2})^2} \text{ along PC} \quad \left[\because BP = \sqrt{r^2+a^2}\right]$$

$$\text{or } \vec{E_1} = \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2+a^2)} \text{ along PC} \quad \text{—— (1)}$$

$$\text{Now } |\vec{E_1}| = \left|\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r^2+a^2)} \text{ along PC}\right|$$

$$\text{or } E_1 = \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2+a^2)} \quad \text{—— (2)}$$

Electric field intensity at P due to charge '−q' is

$$\vec{E_2} = \frac{1}{4\pi\epsilon_0} \frac{q}{(AP)^2} \text{ along PD.}$$

$$\text{or } \vec{E_2} = \frac{1}{4\pi\epsilon_0} \frac{q}{(\sqrt{r^2+a^2})^2} \text{ along PD} \quad \left[\because AP = \sqrt{r^2+a^2}\right]$$

or $\vec{E_2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2+a^2}$ along PD ——— (3)

Now $|\vec{E_2}| = \left| \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2+a^2} \text{ along PD} \right|$

or $E_2 = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2+a^2}$ ——— (4)

From (2) and (4), we get

$E_1 = E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2+a^2}$ ——— (5)

Now electric field intensity $\vec{E_1}$ can be resolved into two rectangular components,

$E_1 \cos\theta$ along PR

and $E_1 \sin\theta$ along PF.

Also electric field intensity $\vec{E_2}$ can be resolved into two rectangular components,

$E_2 \cos\theta$ along PR

and $E_2 \sin\theta$ along PG.

$\because$ Components $E_1 \sin\theta$ and $E_2 \sin\theta$ are equal and opposite therefore, they cancel out. [$\because E_1 = E_2$ $\because$ of (5)]

$\therefore$ Resultant of electric field intensity (E) at P will be vector sum of $E_1 \cos\theta$ and $E_2 \cos\theta$, because these components lie in the same direction.

$\therefore E = E_1 \cos\theta + E_2 \cos\theta$

or $E = E_1 \cos\theta + E_1 \cos\theta$ [$\because E_1 = E_2$ $\because$ of (5)]

or $E = 2 E_1 \cos\theta$ ——— (6)

$E = 2 \times \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2+a^2)} \times \cos\theta$ [$\because$ Putting the value of E_1 from (5) in (6)]

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or $E = \frac{1}{4\pi\epsilon_0} \frac{2q}{(r^2+a^2)} \frac{AO}{AP}$ $\left[\because \text{In } \Delta AOP \quad \cos\theta = \frac{AO}{AP}\right]$

$\therefore E = \frac{1}{4\pi\epsilon_0} \frac{2q}{(r^2+a^2)} \frac{a}{\sqrt{r^2+a^2}}$ $\left[\because AO = a \text{ and } AP = \sqrt{r^2+a^2}\right]$

or $E = \frac{1}{4\pi\epsilon_0} \frac{q \cdot 2a}{(r^2+a^2)^{3/2}}$

or $E = \frac{1}{4\pi\epsilon_0} \frac{p}{(r^2+a^2)^{3/2}}$ —— (7) $\left[\because q \cdot 2a = p \text{ electric dipole moment}\right]$

In vector form,

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\vec{p}}{(r^2+a^2)^{3/2}} \text{ along PB} \quad \text{—— (8)}$$

Special case :—

If $r >> a$, then a^2 can be neglected as compared to r^2 in (7), we get

$$E = \frac{1}{4\pi\epsilon_0} \frac{p}{(r^2)^{3/2}}$$

or $E = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$ —— (9)

In vector form :—

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\vec{p}}{r^3} \text{ along PR} \quad \text{—— (10)}$$

From (9), we get

$$\boxed{E \propto \frac{1}{r^3}}$$

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Q. Find the ratio of the electric field intensity at a point on axial line and at a point at same distance on equitorial line of an electric dipole of very small length.

Sol. Electric field intensity on axial line at a distance r, from the centre of the dipole, is given by

$$E_{axial} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{r^3} \quad \text{---- (1)}$$

Electric field intensity at equitorial line is given by

$$E_{equ} = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3} \quad \text{---- (2)}$$

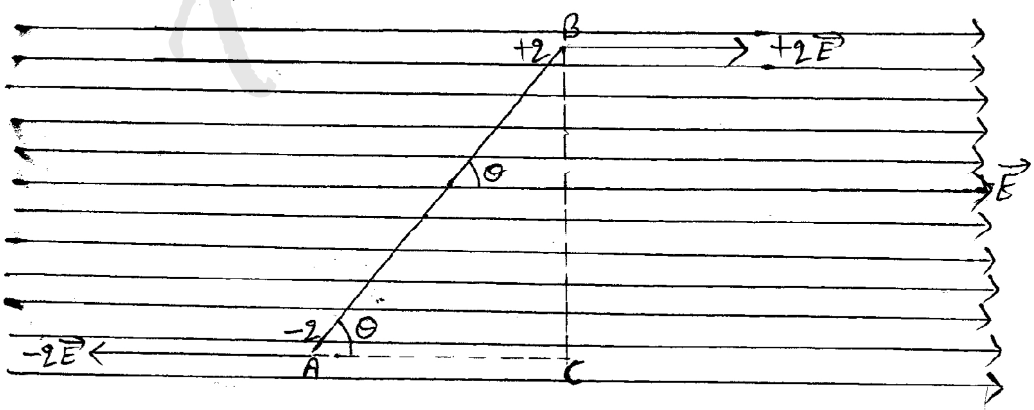
Dividing (1) by (2), we get

$$\frac{E_{axial}}{E_{equ.}} = \frac{\frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{r^3}}{\frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{r^3} \times \frac{4\pi\epsilon_0 \times r^3}{p}$$

$$\Rightarrow \boxed{\frac{E_{axial}}{E_{equ.}} = 2}$$

Electric dipole in a uniform electric field :-

Consider an electric dipole AB having charges $-q$ and $+q$, separated by a distance $2a$, placed in a uniform electric field $\vec{E}$ making an angle θ with the direction of the electric field. Here $AB = 2a$



Force on charge $+q$ at $B = q\vec{E}$ in the direction of electric field $\vec{E}$

Force on charge $-q$ at $A = -q\vec{E}$ in the direction opposite to electric field $\vec{E}$

These forces are equal and opposite constitute a couple and rotate in the clock-wise direction.

$\therefore$ The magnitude of the torque is given by

τ = magnitude of either force $\times$ perpendicular distance between two forces.

$\tau = qE \times BC$

or $\tau = qE \times 2a \sin\theta$

or $\tau = (q \times 2a) E \sin\theta$

or $\tau = pE\sin\theta$ —— (1)

$[\because q \times 2a = p,$ dipole moment$]$

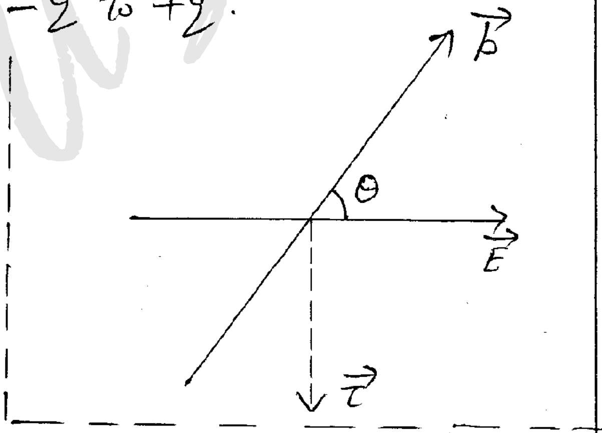
$\left[\because \text{In } \triangle ABC, \ \frac{BC}{AB} = \sin\theta \Rightarrow BC = AB\sin\theta \Rightarrow BC = 2a\sin\theta \ (\because AB = 2a)\right]$

In vector form

$\vec{\tau} = \vec{p} \times \vec{E}$ —— (2)

The direction of $\vec{p}$ is from $-q$ to $+q$.

The direction of $\vec{\tau}$ can be determined by using the right hand screw rule and perpendicular to $\vec{p}$ and $\vec{E}$, directed inwards, as shown in figure:-



Special cases:—

(i) when $\theta = 0°$

Then from equation (1), we get

$\tau = pE\sin 0° = pE(0)$ $[\because \tau = pE\sin\theta]$

$\Rightarrow \tau = 0$ $[\because \sin 0° = 0]$

Thus, when dipole moment $(\vec{p})$ becomes parallel to the electric field $(\vec{E})$, no torque acts on the dipole.

(ii) When $\theta = 180°$

then $\tau = pE\sin 180° = pE(0) = 0$ $[\because \sin 180° = 0]$

$\Rightarrow \tau = 0$

Thus, when dipole moment $(\vec{p})$ is anti-parallel to the electric field $(\vec{E})$, no torque acts on the dipole.

(iii) when $\theta = 90^\circ$

then $\tau = pE \sin 90^\circ$

$\Rightarrow \tau = pE(1) \quad [\because \sin 90^\circ = 1]$

$\Rightarrow \tau = pE$

Thus, when dipole moment $(\vec{p})$ is perpendicular to the electric field $\vec{E}$, maximum torque acts on the dipole.

In magnitude, equation (2) can be written as

$$\tau = p \times E$$

$\therefore$ S.I unit of torque is coulomb metre $\times$ Newton Coulomb^{-1}

= Newton metre

= Nm

$\therefore$ C.g.s unit of torque is stat-Coulomb c.m $\times$ $\frac{\text{dyne}}{\text{stat-Coulomb}}$

= Dyne centimetre

= Dyne c.m.

Dimensional Formula :- τ = force $\times$ distance

$= [MLT^{-2}] \times [L]$

$= [ML^2T^{-2}]$

Potential energy of the electric dipole when placed in uniform electric field :-

The energy possesed by an electric dipole due to its position in an electric field is called potential energy of the dipole.

we know, torque acting on the dipole placed in the uniform electric field is

$$\tau = pE \sin\theta \quad \text{—— (1)}$$

where θ is the angle between dipole moment $(\vec{p})$ and electric field $(\vec{E})$.

Small amount of work done is rotating the dipole through a small angle $d\theta$ against the torque is

$$dw = \text{torque} \times \text{angular displacement}$$

or $dw = \tau \, d\theta$

or $dw = pE \sin\theta \, d\theta$ —— (2) [using (1)]

$\therefore$ Total work done in rotating the dipole from orientation θ_1 to θ_2 is

$$\int dw = \int_{\theta_1}^{\theta_2} pE \sin\theta \cdot d\theta$$

or $w = pE \int_{\theta_1}^{\theta_2} \sin\theta \cdot d\theta = pE \left[-\cos\theta\right]_{\theta_1}^{\theta_2}$ $\left[\because \int \sin\theta \, d\theta = \cos\theta\right]$

or $w = -pE \left[\cos\theta\right]_{\theta_1}^{\theta_2}$

or $w = -pE\left[\cos\theta_2 - \cos\theta_1\right]$ —— (3)

Let $\theta_1 = 90^\circ$ and $\theta_2 = \theta$

$\therefore$ (2) becomes

$$w = -pE\left[\cos\theta - \cos 90^\circ\right] = -pE\left[\cos\theta - 0\right]$$

$\therefore w = -pE\cos\theta$ —— (4) $\left[\because \cos 90^\circ = 0\right]$

This work done is stored in the dipole in the form of energy, which is called potential energy (u) of the dipole.

$\therefore u = -pE\cos\theta$ —— (5)

In vector from, $u = -\vec{p} \cdot \vec{E}$ —— (6)

u is a scalar quantity.

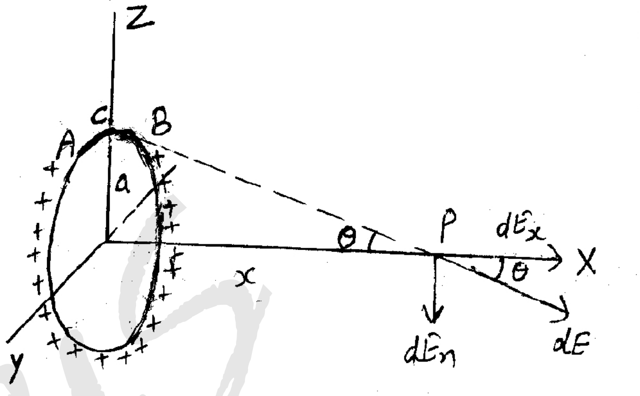
S.I. unit of potential energy is Joule (J).

Electric field due to a circular loop of charge:—

Consider a circular loop of a wire of negligible thick-ness having radius a. Suppose that the total charge q is distributed uniformly over its circumference.

Let the loop is placed in YZ plane with centre at O (origin). The electric field is determined at point P which is at a distance x from the centre of the loop on its axis OX.

Consider an elementary length AB of the loop of length dl.



Then, charge on the elementary length AB is given by:—

$$dq = \frac{q}{2\pi a} dl \quad \text{—— (1)}$$

Let $\vec{dE}$ be electric field at point P due to charge dq on the elementary portion AB of the ring. Then magnitude of electric field at P due to the elementary portion is given by:—

$$dE = |\vec{dE}| = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{CP^2} \quad [\text{Along } CP] \text{ —— (2)}$$

Now $CP = (x^2+a^2)^{1/2}$ —(3) [By taking rt. angled Δ COP $CP^2 = (CO)^2 + (OP)^2$]

By putting values from (1) and (3), in (2), we get

$$dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{q\,dl}{2\pi a(x^2+a^2)}$$

The electric field dE due to elementary portion AB can be resolved into two components:—

(i) dE_x along PX i.e. along X-axis

(ii) dE_n along normal to PX. [For AB, dE_n will be along Z-axis and in the downward direction]

The electric field component dE_n will not contribute to the electric field at point P due to the whole loop, because dE_n due to any two elementary portions of the loop located opposite to each other are equal and oppose and hence cancel out.

∴ Net electric field at P due to the whole loop of charge will be only along x-axis.

If $\vec{dE}$ makes angle θ with x-axis, then

$$dE_x = dE\cos\theta$$

In right angled Δ POC, ∠OPC = θ

$$\therefore \cos\theta = \frac{OP}{PC}$$

$$\text{or} \quad \cos\theta = \frac{x}{\sqrt{(x^2+a^2)}}$$

$$\therefore dE_x = \frac{1}{4\pi\epsilon_0}\cdot\frac{q\,dl}{2\pi a\,(x^2+a^2)}\cdot\frac{x}{(x^2+a^2)^{1/2}}$$

$$\text{or} \quad dE_x = \frac{1}{4\pi\epsilon_0}\cdot\frac{q\,x\,dl}{2\pi a\,(x^2+a^2)^{3/2}}$$

Electric field due to whole circular loop is given by :-

$$E = \int_{\text{total loop}} dE_x = \int_{\text{total loop}} \frac{1}{4\pi\epsilon_0}\cdot\frac{q\,x\,dl}{2\pi a\,(x^2+a^2)^{3/2}}$$

$$= \frac{1}{4\pi\epsilon_0}\cdot\frac{q\,x}{2\pi a\,(x^2+a^2)^{3/2}}\int_{\text{total loop}} dl$$

$$= \frac{1}{4\pi\epsilon_0}\cdot\frac{q\,x\,(2\pi a)}{2\pi a\,(x^2+a^2)^{3/2}}$$

$$E = \frac{1}{4\pi\epsilon_0}\cdot\frac{q\,x}{(x^2+a^2)^{3/2}} \quad [\text{Along PX}] \text{ — } (4)$$

Special cases:

(i) when point P lies at centre of the loop :- In such a case $x=0$. Then from equation (4)

$$E = 0$$

(ii) when point P lies at a distance $x >> a$:- In such a case, In equation (4) a^2 can be neglected in comparison to x^2, then

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{x^2} \quad [\text{Along } PX]$$

It is also the expression for electric field due to a point charge q at a distance x from it.

∴ A circular loop of charge behaves as a point charge, in case the observation point on its axis is at a distance quite large as compared to the radius of the loop.

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