POTENTIAL AND CAPACITANCE

CBSE NCERT Solutions for Class 12 Physics Chapter 2

Back of Chapter Questions

1. Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Solution:

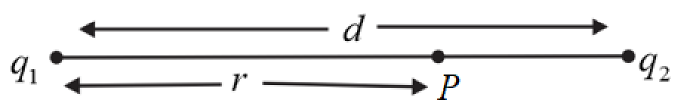
The given two charges are

$q_1 = 5 \times 10^{-8}$ C

$q_2 = -3 \times 10^{-8}$ C

Distance between the two charges is, $d = 16$ cm $= 0.16$ m

Consider on the line joining the two charges, a point P as shown in figure.



Here, r is the distance between point P and charge q_1.

Let the electric potential at point P be zero.

The potential at point P is the sum of the potentials caused by charges q_1 and q_2 respectively.

$\therefore V = \frac{q_1}{4\pi\varepsilon_0 r} + \frac{q_2}{4\pi\varepsilon_0(d-r)}$ …… (i)

Here, ε_0 is the permittivity of free space.

For $V = 0$, equation (i) becomes

$$\frac{q_1}{4\pi\varepsilon_0 r} = -\frac{q_2}{4\pi\varepsilon_0(d-r)}$$

$$\Rightarrow \frac{q_1}{r} = -\frac{q_2}{d-r}$$

$$\Rightarrow \frac{5 \times 10^{-8}}{r} = -\frac{\left(-3 \times 10^{-8}\right)}{(0.16-r)}$$

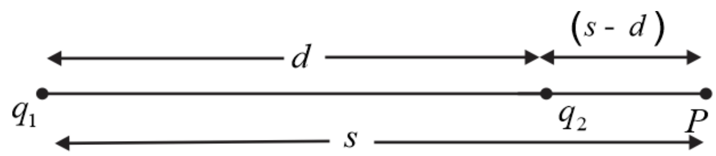
$$\Rightarrow \frac{0.16}{r} - 1 = \frac{3}{5}$$

$$\Rightarrow \frac{0.16}{r} = \frac{8}{5}$$

$\Rightarrow r = 0.1$ m or 10 cm

Thus, the potential is zero at a distance of 10 cm from the positive charge between the charges.

Let there be a point P is outside the system of charges at a distance s from the negative charge, as shown in figure where potential is zero.



The potential, according to the figure is

$$V = \frac{q_1}{4\pi\varepsilon_0 s} + \frac{q_2}{4\pi\varepsilon_0 (s-d)} \qquad \text{...... (ii)}$$

For $V = 0$, equation (ii) becomes

$$\frac{q_1}{4\pi\varepsilon_0 s} = -\frac{q_2}{4\pi\varepsilon_0 (s-d)}$$

$$\Rightarrow \frac{q_1}{s} = -\frac{q_2}{s-d}$$

$$\Rightarrow \frac{5\times 10^{-8}}{s} = -\frac{\left(-3\times 10^{-8}\right)}{(s-0.16)}$$

$$\Rightarrow 1 - \frac{0.16}{s} = \frac{3}{5}$$

$$\Rightarrow \frac{0.16}{s} = \frac{2}{5}$$

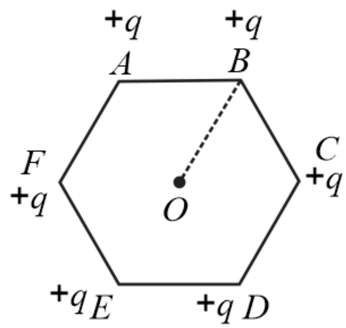
$\Rightarrow s = 0.4$ m or 40 cm

Thus, the potential is zero at a distance of 40 cm from the positive charge outside the system of charges.

2. A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the center of the hexagon.

Solution:

The figure shows the equal amount of charges on the vertices of the hexagon.



The charge is $q = 5\ \mu\text{C} = 5 \times 10^{-6}$ C

The side of the hexagon is $l = \text{AB} = \text{BC} = \text{CD} = \text{DE} = \text{EF} = \text{FA} = 10$ cm

The distance of the charges from the center O is $d = 10$ cm

The electric potential at point O is

$$V = (6)\frac{q}{4\pi\varepsilon_0 d}$$

$$= \frac{6 \times 9 \times 10^9 \times 5 \times 10^{-6}}{0.1}$$

$$= 2.7 \times 10^6 \text{ V}$$

Thus, the potential at the center of the hexagon is 2.7×10^6 V.

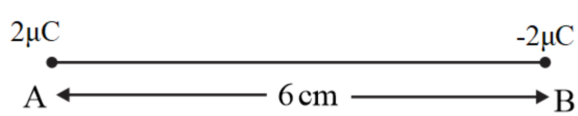
3. Two charges 2 μC and – 2 μC are placed at points A and B 6 cm apart.

(a) Identify an equipotential surface of the system.

(b) What is the direction of the electric field at every point on this surface?

Solution:

(a) The diagram below represents the situation.



An equipotential surface is a plane on which the total potential is zero everywhere. The plane is normal to the line AB. As the magnitude of the charges is the same, the plane is located at the mid-point of line AB.

(b) The direction of the electric field at every point on the equipotential surface is normal to the plane in the direction of AB.

4. A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7} C distributed uniformly on its surface. What is the electric field

(a) inside the sphere

(b) just outside the sphere

(c) at a point 18 cm from the center of the sphere?

Solution:

The radius of the spherical conductor is $r = 12$ cm $= 0.12$ m

The charge is, $q = 1.6 \times 10^{-7}$ C

(a) The electric field inside the sphere is zero because if there is a field inside the conductor, then the charges will move to neutralize it.

(b) The electric field just outside the sphere is

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$= \frac{9 \times 10^9 \times 1.6 \times 10^{-7}}{(0.12)^2}$$

$$= 10^5 \text{ N/C}$$

Thus, the electric field just outside the sphere is 10^5 N/C.

(c) Let the electric field at a point 18 m from the center of the sphere be E_1.

The distance of the point from the center is $d = 18$ cm $= 0.18$ m.

The electric field at a point 18 m from the center of the sphere is,

$$E_1 = \frac{q}{4\pi\varepsilon_0 d^2}$$

$$= \frac{9 \times 10^9 \times 1.6 \times 10^{-7}}{(0.18)^2}$$

$$= 4.4 \times 10^4 \text{ N/C}$$

Thus, the electric field at a point 18 m from the center of the sphere is 4.4×10^4 N/C.

5. A parallel plate capacitor with air between the plates has a capacitance of 8 pF ($1\ pF = 10^{-12}\ F$). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

Solution:

The capacitance between the parallel plate capacitor is, $C = 8$ pF

The dielectric constant of air is, $k = 1$

Let the initial distance between the plates be d.

The capacitance C is

$$C = \frac{k\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d}$$

When the distance is halved, the new distance is $d' = \frac{d}{2}$.

Dielectric constant of the substance is $k' = 6$.

The new capacitance is

$$C' = \frac{k'\varepsilon_0 A}{d'} = \frac{6\varepsilon_0 A}{\left(\frac{d}{2}\right)} = \frac{12\varepsilon_0 A}{d}$$

$$= 12C = 12 \times 8 = 96 \text{ pF}$$

Thus, the capacitance between the plates is 96 pF.

6. Three capacitors each of capacitance 9 pF are connected in series.

(a) What is the total capacitance of the combination?

(b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

Solution:

(a) The capacitance of each capacitance is $C = 9$ pF

The equivalent capacitance of the combination (C_{eq}) is given by the relation,

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$$

$$C_{eq} = 3\ \mu\text{F}$$

Thus, the equivalent capacitance of the combination is 3 μF.

(b) The supply voltage is 100 V.

The potential difference across each capacitor is one-third of the supply voltage.

$$V' = \frac{V}{3} = \frac{120 \text{ V}}{3} = 40 \text{ V}$$

Thus, the potential difference across each capacitor is 40 V.

7. Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.

(a) What is the total capacitance of the combination?

(b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.

Solution:

(a) The capacitances of the capacitors are

$C_1 = 2 \text{ pF}$

$C_2 = 3 \text{ pF}$

$C_3 = 4 \text{ pF}$

The equivalent capacitance of the parallel combination $(C_{eq})$is given by the relation,

$C_{eq} = C_1 + C_2 + C_3 = 2 + 3 + 4 = 9 \text{ pF}$

Thus, the equivalent capacitance of the parallel combination is 9 μF.

(b) The supply voltage is 100 V.

The potential difference across each capacitor is the same as they are connected in parallel.

The relation between the charge on a capacitor and the potential difference is,

$q = VC$

The charge on the first capacitor is,

$q_1 = VC_1 = 100 \times 2 = 200 \text{ pF} = 2 \times 10^{-10} \text{ C}$

The charge on the second capacitor is,

$q_2 = VC_2 = 100 \times 3 = 300 \text{ pF} = 3 \times 10^{-10} \text{ C}$

The charge on the third capacitor is,

$q_3 = VC_3 = 100 \times 4 = 400 \text{ pF} = 4 \times 10^{-10} \text{ C}$

8. In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3} \text{ m}^2$ and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

Solution:

Area of the plate is, $A = 6 \times 10^{-3} \text{ m}^2$

Distance between the plates, $d = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$

The supply voltage is, $V = 100 \text{ V}$

The capacitance of the parallel plate capacitor is,

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}}$$

$\Rightarrow C = 17.71 \times 10^{-12}$ F or 17.71 pF

The charge on each plate of the capacitor is,

$q\ = VC$

$= 100 \times 17.71 \times 10^{-12}$

$= 1.771 \times 10^{-9}$ C

Thus, the capacitance of the capacitor is 17.71 pF and the charge on each plate is 1.771×10^{-9} C.

9. Explain what would happen if in the capacitor given in Exercise 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,

(a) while the voltage supply remained connected.

(b) after the supply was disconnected.

Solution:

(a) The thickness of the mica sheet, 3 mm

The dielectric constant of the mica sheet, $k = 6$

The initial capacitance, $C = 1.771 \times 10^{-11}$ F

The new capacitance is,

$C' = kC$

$= 6 \times 1.771 \times 10^{-11}$ F

$= 106 \times 10^{-12}$ F or 106 pF

The supply voltage is 100 V.

Thus, the new charge while the voltage supply remained connected is,

$q' = C'V$

$= 106 \times 10^{-12} \times 100$

$= 1.06 \times 10^{-8}$ C

Thus, the new charge while the voltage supply remained connected is 1.06×10^{-8} C.

(b) When the supply is disconnected, the charge will be constant. Thus, the potential across the plates is,

$$V' = \frac{q'}{C'} = \frac{1.771 \times 10^{-9}\ \mathrm{C}}{106 \times 10^{-12}\ \mathrm{F}} = 16.7\ \mathrm{V}$$

Thus, the potential across the plate when the supply was disconnected is 16.7 V.

10. A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor?

Solution:

The capacitance of the capacitor is, $C = 12\ \mathrm{pF} = 12 \times 10^{-12}\ \mathrm{F}$

The potential difference across the capacitor is, $V = 50\ \mathrm{V}$

Thus, the electrostatic energy stored in the capacitor is,

$$E = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times 12 \times 10^{-12}\ \mathrm{F} \times (50\ \mathrm{V})^2$$

$$= 1.5 \times 10^{-8}\ \mathrm{J}$$

Thus, the electrostatic energy stored in the capacitor is 1.5×10^{-8} J.

11. A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

Solution:

The capacitance of the capacitor is, $C = 600\ \mathrm{pF} = 600 \times 10^{-12}\ \mathrm{F}$

The potential difference is, $V = 200\ \mathrm{V}$

Electrostatic energy stored in the capacitor is,

$$E = \frac{1}{2}CV^2$$

$$= \frac{1}{2}(600 \times 10^{-12})(200)^2$$

$$= 1.2 \times 10^{-5}\ \mathrm{J}$$

If the supply is disconnected from the capacitor, and another capacitor is connected to it, then the equivalent capacitance is,

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} = \frac{1}{600} + \frac{1}{600} = \frac{2}{600} = \frac{1}{300}$$

$$\Rightarrow C' = 300\ \mathrm{pF}$$

The new electrostatic energy is,

$$E' = \frac{1}{2}C'V^2 = \frac{1}{2}(300 \times 10^{-12})(200)^2 = 0.6 \times 10^{-5} \text{ J}$$

The amount of electrostatic energy lost is

$$E - E' = 1.2 \times 10^{-5} - 0.6 \times 10^{-5}$$

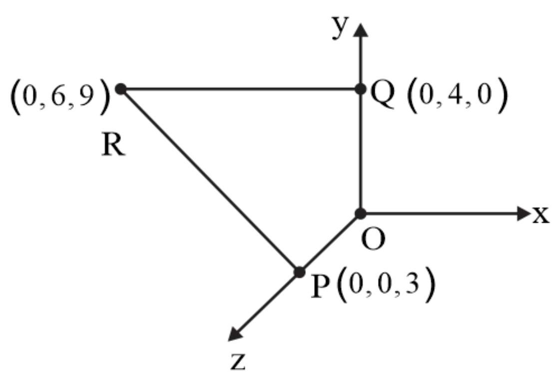
$$= 6 \times 10^{-6} \text{ J}$$

Thus, the electrostatic energy lost in the process is 6×10^{-6} J.

12. A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of $-2 \times 10-9\ C$ from a point P $(0, 0, 3\text{ cm})$ to a point Q $(0, 4\text{ cm}, 0)$, via a point R $(0, 6\text{ cm}, 9\text{ cm})$.

Solution:

The points are shown in the figure below.



The charge that is located at the origin is, $q = 8\text{ mC} = 8 \times 10^{-3}\text{ C}$

The magnitude of the small charge that is moved from P to Q is, $q_1 = 2 \times 10^{-9}\text{ C}$

Point P is at a distance, $d_1 = 3$ cmfrom the origin along the z-axis.

Point Q is at a distance, $d_2 = 4$ cmfrom the origin along the y-axis.

The potential at point P is, $V_1 = \frac{q}{4\pi\varepsilon_0 d_1}$

The potential at point Q is, $V_2 = \frac{q}{4\pi\varepsilon_0 d_2}$

The work done by the electrostatic force is path independent.

$$W = q_1[V_2 - V_1]$$

$$= q_1\left[\frac{q}{4\pi\varepsilon_0 d_2} - \frac{q}{4\pi\varepsilon_0 d_1}\right]$$

$$= \frac{qq_1}{4\pi\varepsilon_0}\left[\frac{1}{d_2} - \frac{1}{d_1}\right]$$

Therefore, the work done is,

$$W = 9 \times 10^{-9} \times 8 \times 10^{-3} \times (-2 \times 10^{-9})\left[\frac{1}{0.04} - \frac{1}{0.03}\right]$$

$$= -144 \times 10^{-3} \times \left(\frac{-25}{3}\right) = 1.27 \text{ J}$$

Thus, the work done during the process is 1.27 J.

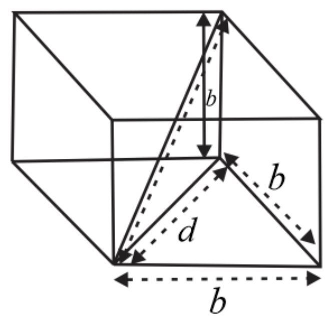
13. A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the center of the cube.

Solution:

The length of the side of the cube is b.

The length of the vertices is q.

The cube is shown in the figure below.



The diagonal of one of the six faces of the cube d is given by

$$d = \sqrt{b^2 + b^2} = \sqrt{2b^2} = b\sqrt{2}$$

The length of the diagonal of the cube is given by

$$l^2 = \sqrt{d^2 + b^2}$$

$$= \sqrt{\left(\sqrt{2}b\right)^2 + b^2} = \sqrt{2b^2 + b^2} = \sqrt{3b^2}$$

$$= b\sqrt{3}$$

The distance between the center of the cube and one of the eight vertices is,

$$r = \frac{l}{2} = \frac{b\sqrt{3}}{2}$$

The electric potential at the center of the cue is due to the eight charges. Thus, the net electric potential is,

$$V=\frac{8q}{4\pi\varepsilon_0 r}=\frac{8q}{4\pi\varepsilon_0\left(b\frac{\sqrt{3}}{2}\right)}=\frac{4q}{\sqrt{3}\pi\varepsilon_0 b}$$

Thus, the potential at the center of the cube is $V=\frac{4q}{\sqrt{3}\pi\varepsilon_0 b}$.

As the charges are distributed symmetrically with respect to the center of the cube, the electric field due to the eight charges get cancelled. Thus, the electric field at the center of the cube is zero.

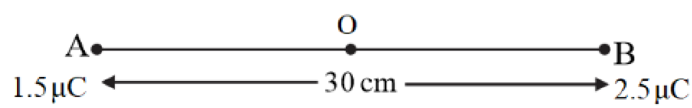
14. Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field:

(a) at the mid-point of the line joining the two charges, and

(b) at a point 10 cm from this midpoint in a plane normal to the line and passing through the mid-point.

Solution:

The two charges are placed as shown in figure.



Let O be the mid-point of the line joining the charges.

The magnitude of the charge at point A is $q_1 = 1.5\ \mu\text{C}$.

The magnitude of the charge at point B is $q_2 = 2.5\ \mu\text{C}$.

The distance between the two charges, $d = 30\text{ cm} = 0.3\text{ m}$

(a) Let the electric field and electric potential at point O respectively be E_1 and V_1.

The electric potential at point O is the sum of the potential due to charge at A and B.

$$V_1=\frac{q_1}{4\pi\varepsilon_0\left(\frac{d}{2}\right)}+\frac{q_2}{4\pi\varepsilon_0\left(\frac{d}{2}\right)}$$

$$=\frac{1}{4\pi\varepsilon_0\left(\frac{d}{2}\right)}(q_1+q_2)$$

$$= \frac{9 \times 10^9 \times 10^{-6}}{\left(\frac{0.30}{2}\right)}(2.5 + 1.5)$$

$$= 2.4 \times 10^5 \text{ V}$$

Electric field at point O is

E_1 = Electric field due to q_2 − Electric field due to q_1

$$E_1 = \frac{q_2}{4\pi\varepsilon_0\left(\frac{d}{2}\right)^2} - \frac{q_1}{4\pi\varepsilon_0\left(\frac{d}{2}\right)^2}$$

$$= \frac{1}{4\pi\varepsilon_0\left(\frac{d}{2}\right)^2}(q_2 - q_1)$$

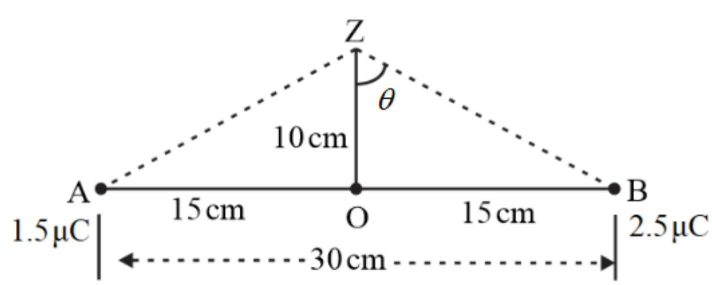
$$= \frac{9 \times 10^9 \times 10^{-6}}{\left(\frac{0.30}{2}\right)^2}(2.5 - 1.5)$$

$$= 4 \times 10^5 \text{ V/m}$$

Thus, the electric potential at the midpoint is 2.4×10^5 V and the electric field at the midpoint is 4×10^5 V/m. The direction of the field is from the larger charge to the smaller charge.

(b) Consider a point Z such that the distance OZ is OZ = 10 cm = 0.1 m.

The figure is shown below.



Let the electric field and electric potential at point Z respectively be E_2 and V_2.

From the figure, the distance BZ = AZ = $\sqrt{(0.1)^2 + (0.15)^2}$ = 0.18 m.

The electric potential at point Z is the sum of the potentials due to charges at A and B.

$$V_2 = \frac{q_1}{4\pi\varepsilon_0(\text{AZ})} + \frac{q_2}{4\pi\varepsilon_0(\text{BZ})}$$

$$= \frac{1}{4\pi\varepsilon_0(0.18)}(q_1 + q_2)$$

$$= \frac{9 \times 10^9 \times 10^{-6}}{0.18}(2.5 + 1.5)$$

$$= 2 \times 10^5 \text{ V}$$

Electric field due to q_1 at point Z is

$$E_A = \frac{q_1}{4\pi\varepsilon_0(\text{AZ})^2}$$

$$= \frac{9 \times 10^9 \times 1.5 \times 10^{-6}}{0.18^2}$$

$$= 0.416 \times 10^6 \text{ V/m}$$

Electric field due to q_2 at point Z is

$$E_B = \frac{q_2}{4\pi\varepsilon_0(\text{BZ})^2}$$

$$= \frac{9 \times 10^9 \times 2.5 \times 10^{-6}}{0.18^2}$$

$$= 0.69 \times 10^6 \text{ V/m}$$

The angle is,

$$\cos\theta = \frac{0.10}{0.18} = 0.5556$$

$$\theta = \cos^{-1}(0.5556) = 56.25$$

$$2\theta = 112.5^{\circ}$$

$$\cos 2\theta = -0.38$$

The resultant field intensity at Z is,

$$E = \sqrt{E_A^2 + E_B^2 + 2E_AE_B\cos 2\theta}$$

$$= \sqrt{(0.416 \times 10^6)^2 + (0.69 \times 10^6)^2 + 2(0.416 \times 10^6)(0.69 \times 10^6)(-0.38)}$$

$$= 6.6 \times 10^5 \text{ V/m}$$

Thus, the electric potential at a point 10 cm is 2×10^5 V and the electric field at a point 10 cm is 6.6×10^5 V/m.

15. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q.

(a) A charge q is placed at the center of the shell. What is the surface charge density on the inner and outer surfaces of the shell?

(b) Is the electric field inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape? Explain.

Solution:

(a) When a charge q is placed at the center of a shell, a charge of $-q$ will be induced to the inner surface of the shell. Thus, the inner surface of the shell will have a total charge of $-q$.

Surface charge density at the inner surface of the shell is given by

$$\sigma_1 = \frac{\text{total charge}}{\text{inner surface area}} = \frac{-q}{4\pi r_1^2}$$

A charge $+q$ is induced on the outer surface of the shell. Also, the outer surface has a charge of Q on it. Thus, the total charge on the outer surface of the shell is $Q + q$.

Surface charge density at the outer surface of the shell is given by

$$\sigma_2 = \frac{\text{total charge}}{\text{outer surface area}} = \frac{Q+q}{4\pi r_2^2}$$

(b) The electric field inside a cavity is zero, even if the shell is not spherical, but has any irregular shape. As an example, consider a closed loop with part of it inside the cavity along the field and the rest is inside the conductor. As the field inside the conductor is zero, the net work done by the field in carrying a test charge over the closed loop is zero. Thus, the electric field is zero irrespective of the shape.

16. (a) Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by $\left(\vec{E}_2 - \vec{E}_1\right) \cdot \hat{n} = \frac{\sigma}{\varepsilon_0}$ where $\hat{n}$ is a unit vector normal to the surface at a point and s is the surface charge density at that point. (The direction of $\hat{n}$ is from side 1 to side 2.) Hence, show that just outside a conductor, the electric field is $\frac{\sigma \hat{n}}{\varepsilon_0}$.

(b) Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another.

[Hint: For (a), use Gauss's law. For, (b) use the fact that work done by electrostatic field on a closed loop is zero.]

Solution:

(a) The electric field on one side of a charged body is E_1 and electric field on the other side of the body is E_2. The electric field due to one surface of the charged body if infinite plane charged body has a uniform thickness is,

$$\overrightarrow{E_1} = -\frac{\sigma}{2\varepsilon_0}\,\hat{n} \;\ldots(i)$$

Here,

$\hat{n}$ = Unit vector normal to the surface at a point.

σ = Surface charge density at the point.

Due to the other surface of the charged body, the electric field is,

$$\overrightarrow{E_2} = -\frac{\sigma}{2\varepsilon_0}\,\hat{n} \;\ldots(ii)$$

The electric field at any point due to the two surfaces,

$$\overrightarrow{E_2} - \overrightarrow{E_1} = -\frac{\sigma}{2\varepsilon_0}\,\hat{n} + \frac{\sigma}{2\varepsilon_0}\,\hat{n} = \frac{\sigma}{\varepsilon_0}\,\hat{n}$$

$$\left(\overrightarrow{E_2} - \overrightarrow{E_1}\right).\hat{n} = \frac{\sigma}{\varepsilon_0} \;\ldots(iii)$$

The electric field inside a closed conductor is zero.

Thus, $\overrightarrow{E_1} = 0$

$$\therefore \vec{E} - \overrightarrow{E_2} = -\frac{\sigma}{2\varepsilon_0}\,\hat{n}$$

Thus, the electric field just outside the conductor is $\frac{\sigma}{\varepsilon_0}\,\hat{n}$.

The work done by the electrostatic field is zero when a charged particle is moved from one point to the other on a closed loop. Thus, the tangential component of electrostatic field is continuous from one side of a charged surface to another.

17. A long charged cylinder of linear charged density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?

Solution:

The charge density of the long charged cylinder is λ. Let the radius if the hollow cylinder be R. If E is the electric field between the cylinders, then according to Gauss's theorem, the electric flux through the Gaussian surface is,

$$\phi = E(2\pi d)L$$

Here, d is the distance of a point from the common axis of the cylinders.

If q is the total charge on the cylinder, then

$$\therefore\; \phi = E(2\pi dL) = \frac{q}{\varepsilon}$$

$$E(2\pi dL) = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 d}$$

Here,

q = Charge on the inner sphere of the outer cylinder.

ε_0 = Permittivity of free space.

Thus, the electric field in the space between the two cylinders is $\frac{\lambda}{2\pi\,\varepsilon_0 d}$.

18. In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 Å:

(a) Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton.

(b) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (a)?

(c) What are the answers to (a) and (b) above if the zero of potential energy is taken at 1.06 Å separation?

Solution:

Given

The distance between the electron and proton in a hydrogen atom is, $d = 0.53$ Å.

(a) The potential at infinity is zero.

The potential energy of the system is the difference between the potential energy at infinity and potential energy at d.

Potential energy of the system $= 0 - \frac{q_1 q_2}{4\pi\varepsilon_0\, d}$

Potential energy $= 0 - \frac{9\times10^{9}\times(1.6\times10^{-19})^2}{0.53\times10^{10}} = -43.7\times10^{-19}$ J

Since 1.6×10^{-19}J $= 1$ eV,

$\therefore$ Potential energy $= -43.7\times10^{-19} = \frac{-43.7\times10^{-19}}{1.6\times10^{-19}} = -27.2$ eV

Thus, the potential energy of the system is -27.2 eV.

(b) As the kinetic energy is half the potential energy,

$$\text{KE} = \frac{1}{2}(-27.2) = 13.6 \text{ eV}$$

Total energy $= 13.6 - 27.2 = 13.6$ eV

(c) The potential of the system is the difference between potential energy at d_1 and potential at d.

Potential of the system

$$= \frac{q_1 q_2}{4\pi\varepsilon_0 d_1} - 27.2 \text{ eV}$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{1.06 \times 10^{-10}} = -27.2 \text{ eV}$$

$$= 21.73 \times 10^{-19} \text{ J} - 27.2 \text{ eV}$$

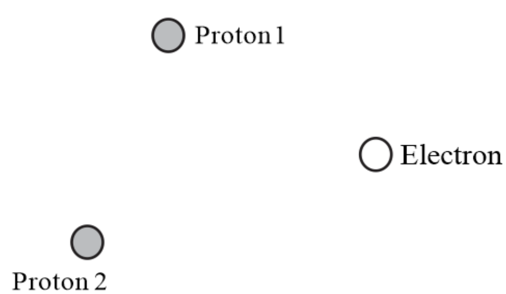
$$= 13.58 \text{ eV} - 27.2 \text{ eV}$$

$$= -13.6 \text{ eV}$$

19. If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion H_2^+. In the ground state of an H_2^+, the two protons are separated by roughly 1.5 Å, and the electron is roughly 1 Å from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.

Solution:

The diagram showing the system with two protons and an electron is shown below.



Given

The distance between proton 1 and proton 2 is $d_1 = 1.5 \times 10^{-10}$ m.

Distance between proton 1 and electron, $d_2 = 1 \times 10^{-10}$ m.

Distance between proton 2 and electron, $d_3 = 1 \times 10^{-10}$ m.

The total potential energy of the system is,

$$V = \frac{q_1 q_2}{4\pi\varepsilon_0 d_1} + \frac{q_2 q_3}{4\pi\varepsilon_0 d_3} + \frac{q_3 q_1}{4\pi\varepsilon_0 d_2}$$

$$= \frac{9 \times 10^9 \times 10^{-19} \times 10^{-19}}{10^{-10}} \left[(-16)^2 + \frac{(1.6)^2}{1.5} + -(1.6)^2\right]$$

$= -30.7 \times 10^{-19}$ J

$= -19.2$ eV

Thus, the potential energy of the system is -19.2 eV.

20. Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.

Solution:

Assume a to be the radius of a sphere A, Q_A be the charge on the sphere, and C_A be the capacitance of the sphere.

Let b be the radius of a sphere B, Q_B be the charge on the sphere, and C_B be the capacitance of the sphere. Since the two spheres are connected with a wire, their potential (V) will become equal.

Let E_A be the electric field of sphere A and E_B be the electric field of sphere B.

Thus, their ratio,

$$\frac{E_A}{E_B} = \frac{Q_A}{4\pi\,\varepsilon_0 \times a_2} \times \frac{b^2 \times 4\pi\,\varepsilon_0}{Q_B}$$

$\Rightarrow \frac{E_A}{E_B} = \frac{Q_A}{Q_B} \times \frac{b^2}{b^2}$ … (1)

We know, $\frac{Q_A}{Q_B} = \frac{C_A V}{C_B V}$ and, $\frac{C_A}{C_B} = \frac{a}{b}$

$\frac{Q_A}{Q_B} = \frac{a}{b}$ …. (2)

Putting the value of (2) in (1), we obtain

$$\therefore \frac{E_A}{E_B} = \frac{a}{b}\frac{b^2}{a^2} = \frac{b}{a}$$

Thus, the ratio of electric fields at the surface is $\frac{b}{a}$.

21. Two charges $-q$ and $+q$ are located at points $(0, 0, -a)$ and $(0, 0, a)$, respectively.

(a) What is the electrostatic potential at the points $(0, 0, z)$ and $(x, y, 0)$?

(b) Obtain the dependence of potential on the distance r of a point from the origin when $r/a >> 1$.

(c) How much work is done in moving a small test charge from the point $(5, 0, 0)$ to $(-7, 0, 0)$ along the x-axis? Does the answer change if the path of the test charge between the same points is not along the x-axis?

Solution:

(a) Charge $-q$ is located at $(0, 0, -a)$ and charge $+q$ is located at $(0, 0, a)$. Thus, the charges will form a dipole.

The point $(0, 0, z)$ is on the axis of this dipole and point $(x, y, 0)$ is normal to the axis of the dipole. Hence, electrostatic potential at point $(x, y, 0)$ is zero.

(b) As the distance r is greater than half the distance between the two charges, the potential at a distance r is inversely proportional to the square of the distance.

Thus, $V \propto \frac{1}{r^2}$.

(c) The work done in moving a small test charge from the point $(5, 0, 0)$ to $(-7, 0, 0)$ along the x-axis is zero.

The electrostatic potential (V_1) at point $(5, 0, 0)$ is,

$$V_1 = \frac{-q}{4\pi\varepsilon_0} \frac{1}{\sqrt{(5-0)^2 + (-a)^2}} + \frac{q}{4\pi\varepsilon_0} \frac{1}{(5-0)^2 + a^2}$$

$$= \frac{-q}{4\pi\varepsilon_0\sqrt{25^2 + a^2}} + \frac{q}{4\pi\varepsilon_0\sqrt{25 + a^2}}$$

$$= 0$$

The electrostatic potential (V_2), at point $(-7, 0, 0)$ is given by,

$$V_2 = \frac{-q}{4\pi\varepsilon_0} \frac{1}{\sqrt{(-7)^2 + (-a)^2}} + \frac{q}{4\pi\varepsilon_0} \frac{1}{\sqrt{(-7)^2 + a^2}}$$

$$= \frac{-q}{4\pi\varepsilon_0\sqrt{49 + a^2}} + \frac{q}{4\pi\varepsilon_0} \frac{1}{\sqrt{49 + a^2}}$$

$$= 0$$

Thus, no work is done in moving a small test charge from the point $(5, 0, 0)$ to $(-7, 0, 0)$ along the x-axis.

Even if the path is not along x-axis, the answer does not change because the work done is independent of the path.

22. Figure 2.32 shows a charge array known as an *electric quadrupole*. For a point on the axis of the quadrupole, obtain the dependence of potential on r for $r/a >> 1$, and contrast your results with that due to an electric dipole, and an electric monopole (i.e., a single charge).

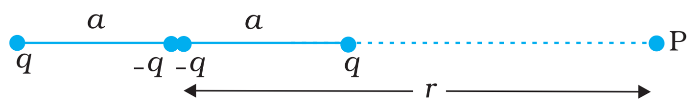
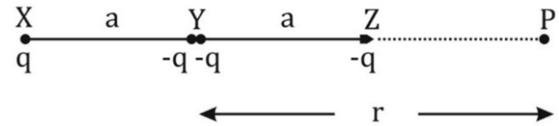


FIGURE 2.34

Solution:

The figure showing the four charges is shown below.



The distance between point P and Y is r.

The system can be considered as a system with three charges,

Charge $+q$ placed at point X.

Charge $-2q$ placed at point Y.

Charge $+q$ placed at point Z.

The distances XY $=$ YZ $= a$, YP $= r$, PX $= r + a$, PZ $= r - a$.

Electrostatic potential caused by the system of three charges is,

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{XP} - \frac{2q}{YP} + \frac{q}{ZP}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r+a} - \frac{2q}{r} + \frac{q}{r-a}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r(r-a) - 2(r+a)(r-a) + r(r+a)}{r(r+a)(r-a)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2 - ra - 2r^2 + 2a^2 + r^2 + ra}{r(r^2 - a^2)}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{2a^2}{r(r^2 - a^2)}\right]$$

$$= \frac{2qa^2}{4\pi\varepsilon_0\, r^3\left(1 - \frac{a^2}{r^2}\right)}$$

As, $\frac{r}{a} \gg 1$

$\therefore \frac{a}{r} \ll 1$ and $\frac{a^2}{r^2}$ is taken as negligible.

$$\therefore V = \frac{2qa^2}{4\pi\varepsilon_0 r^3}$$

Thus, it can be concluded that potential, $V \propto \frac{1}{r^3}$

And, for a dipole, $V \propto \frac{1}{r^2}$

And, for a monopole, $V \propto \frac{1}{r}$

23. An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors.

Solution:

Given,

The required capacitance, $\mathrm{C} = 2$ μF.

The potential difference, $V = 1$ kV $= 1000$ V.

The maximum potential difference each capacitance can withstand is, $V_1 = 400$ V.

Assume a number of capacitors are connected in series and these series circuits are connected in parallel (row) to each other.

The potential difference across each row must be 1000 V. And the potential difference across each capacitor must be 400 V. Hence, the number of capacitors in each row is given as

$$\frac{1000}{400} = 2.5.$$

Hence, it can be concluded that there are three capacitors in each row.

Capacitance of each row $= \frac{1}{1+1+1} = \frac{1}{3}$ μF

Let there be n rows, each having three capacitors connected in parallel.

Hence, equivalent capacitance of the circuit is given as

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \cdots n \text{ terms}$$

$$= \frac{n}{3}$$

But, capacitance of the circuit is given as 2 μF.

$$\therefore \frac{n}{3} = 2$$

$$n = 6$$

Hence, 6 rows of three capacitors are present in the circuit.

A minimum of 6 × 3 i.e., 18 capacitors are required for the given arrangement.

24. What is the area of the plates of a 2 F parallel plate capacitor, given that the separation between the plates is 0.5 cm? [You will realize from your answer why ordinary capacitors are in the range of µF or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductors.]

Solution:

Given,

The capacitance is $C = 2$ F.

The plate separation is, $d = 0.5\text{ cm} = 0.5 \times 10^{-2}$ m.

The capacitance of a parallel plate capacitor is given by the relation,

$$C = \frac{\varepsilon_0 A}{d}$$

$$A = \frac{CV}{\varepsilon_0}$$

$$= \frac{2 \times 0.5 \times 10^{-2}}{8.85 \times 10^{-12}}$$

$$= 1129943502\text{ m}^2$$

$$= 1130\text{ km}^2$$

Thus, the area of the capacitors is too large.

25. Obtain the equivalent capacitance of the network in Figure 2.35. For a 300 V supply, determine the charge and voltage across each capacitor.

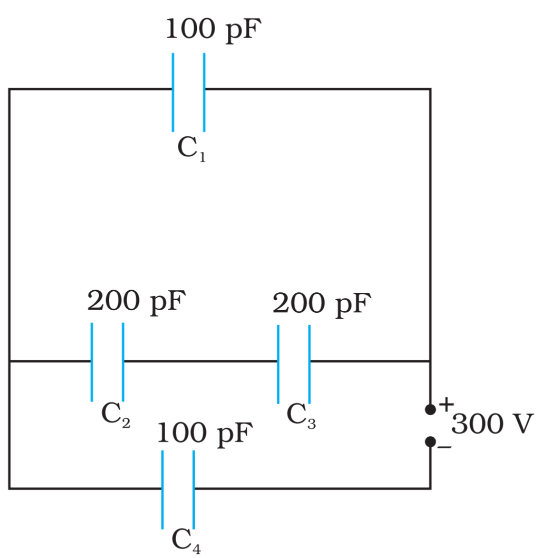


FIGURE 2.35

Solution:

The capacitors C_2 and C_3 are connected in series. The resultant is,

$$\frac{1}{C'} = \frac{1}{200} + \frac{1}{200} = \frac{2}{200}$$

$C' = 100$ pF

As the capacitors C_1 and C' are parallel, the equivalent capacitance is

$C" = C' + C_1$

$= 100 + 100 = 200$ pF

As the capacitors $C"$ and C_4 are series, the equivalent capacitance is

$$\frac{1}{C} = \frac{1}{C"} + \frac{1}{C_4}$$

$$= \frac{1}{200} + \frac{1}{100} = \frac{3}{200}$$

$$C = \frac{200}{3} \text{ pF}$$

Thus, the equivalent capacitance of the circuit is $\frac{200}{3}$ pF.

Let the potential difference across $C”$ be $V”$ and the potential difference across C_4 be V_4

$\therefore V" + V_4 = V = 300$ V

The charge of C_4 is given by

$$Q_4 = CV = \frac{200}{3} \times 10^{-12} \times 300 = 2 \times 10^{-8} \text{ C}$$

Thus, the potential across C_4 is,

$$V_4 = \frac{Q_4}{C_4} = \frac{2 \times 10^{-8}}{100 \times 10^{-12}} = 200 \text{ V}$$

The potential across C_1 is,

$V_1 = V - V_4 = 300 - 200 = 100$ V

The charge on C_1 is,

$Q_1 = C_1 V_1 = 100 \times 10^{-12} \times 100 = 10^{-8}$ C

As C_2 and C_3 have the same capacitance, the potential difference across them is 100 V together.

As C_2 and C_3 are in series, the potential difference across them is

$V_2 = V_3 = 50\text{ V}$

The charge on C_2 is,

$Q_2 = C_2V_2 = 200 \times 10^{-12} \times 50 = 10^{-8}\text{ C}$

The charge on $C_3$is,

$Q_3 = C_3V_3 = 200 \times 10^{-12} \times 50 = 10^{-8}\text{ C}$

Thus, the equivalent capacitance of the circuit is $\frac{200}{3}$ pF.

The charge across capacitance C_1 is 10^{-8} C and the potential difference is 100 V.

The charge across capacitance C_2 is 10^{-8} C and the potential difference is 50 V.

The charge across capacitance C_3 is 10^{-8} C and the potential difference is 50 V.

The charge across capacitance C_4 is 2×10^{-8} C and the potential difference is 200 V.

26. The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.

(a) How much electrostatic energy is stored by the capacitor?

(b) View this energy as stored in the electrostatic field between the plates, and obtain the energy per unit volume u. Hence arrive at a relation between u and the magnitude of electric field E between the plates.

Solution:

The area of the parallel plate capacitor, $A = 90\text{ cm}^2 = 90 \times 10^{-4}\text{ m}^2$

Distance between the plates, $d = 2.5\text{ mm} = 1.5 \times 10^{-3}\text{ m}$

Potential difference across the plates, $V = 400\text{ V}$

(a) The capacitance of the capacitor is given by $C = \frac{\varepsilon_0 A}{d}$.

Electrostatic energy stored in the capacitor is given by $E_1 = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2$.

Substituting the values,

$$E_1 = \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2$$

$$= \frac{1 \times 8.85 \times 10^{-12} \times 90 \times 10^{-4} \times (400)^2}{2 \times 2.5 \times 10^{-3}}$$

$$= 2.55 \times 10^{-6}\text{ J}$$

Thus, the electrostatic energy stored in the capacitor is 2.55×10^{-6} J.

(b) The volume of the capacitor is,

$V' = Ad$

$= 90 \times 10^{-4} \times 25 \times 10^{-3}$

$= 2.25 \times 10^{-4}\ \text{m}^3$

Energy stored in the electrostatic field between the plates per unit volume is

$$u = \frac{E_1}{V'} = \frac{2.55 \times 10^{-6}}{2.23 \times 10^{-4}} = 0.113\ \text{J/m}^3$$

Another equation for energy per unit volume is,

$$u = \frac{E_1}{V'}$$

$$= \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{\varepsilon_0 A}{2d}V^2}{Ad} = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_0 E^2$$

Thus, the relation between u and the magnitude of electric field E between the plates is $\frac{1}{2}\varepsilon_0 E^2$.

27. A 4 μF capacitor is charged by a 200 V supply. It is then disconnected the supply and is connected to another uncharged 2 μF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?

Solution:

The capacitance of the charged capacitor is,

$C_1 = 4\ \mu\text{F} = 4 \times 10^{-6}$ F

The supply voltage is, 200 V

The electrostatic energy stored in the charged capacitor is,

$$E_1 = \frac{1}{2}C_1 V_1^2$$

$$= \frac{1}{2}(4 \times 10^{-6}\ \text{F})(200\ \text{V})^2 = 8 \times 10^{-2}\ \text{J}$$

The capacitance of the uncharged capacitor is,

$C_2 = 2\ \mu\text{F} = 2 \times 10^{-6}$ F

When the uncharged capacitor is connected to the circuit, there will be a potential difference across the capacitor. Since the charge is conserved, the final charge on the charged and uncharged capacitors together will be the initial charge of the charged capacitor. Therefore,

$$C_1V_1 = (C_1 + C_2)V_2$$

Here, V_2 is the potential difference across the uncharged capacitor.

Therefore, the potential difference across the uncharged capacitor is,

$$V_2 = \frac{C_1}{(C_1 + C_2)}V_1$$

$$= \frac{4 \times 10^{-6}\text{ F}}{(4 \times 10^{-6}\text{ F} + 2 \times 10^{-6}\text{ F})} \times 200\text{ V}$$

$$= 1.33 \times 10^2\text{ V}$$

The electrostatic potential energy stored in the combination of capacitors is,

$$E_2 = \frac{1}{2}(C_1 + C_2)V_2^2$$

$$= \frac{1}{2}(4 \times 10^{-6}\text{ F} + 2 \times 10^{-6}\text{ F})(1.33 \times 10^2\text{ V})^2$$

$$= 5.33 \times 10^{-2}\text{ J}$$

Therefore, the energy lost by charged capacitor is,

$$\Delta E = E_1 - E_2$$

$$= 8 \times 10^{-2}\text{ J} - 5.33 \times 10^{-2}\text{ J}$$

$$= 2.67 \times 10^{-2}\text{ J}$$

Thus, the energy lost by first capacitor as heat and electromagnetic radiation is 2.67×10^{-2} J.

28. Show that the force on each plate of a parallel plate capacitor has a magnitude equal to $\left(\frac{1}{2}\right) QE$, where Q is the charge on the capacitor, and E is the magnitude of electric field between the plates. Explain the origin of the factor $\frac{1}{2}$.

Solution:

The work done to separate a parallel plate capacitor by applying a force F to a distance of Δx is,

$$W = F\Delta x$$

Therefore, the increase in potential energy of the capacitor is,

$$\Delta E = UA\Delta x$$

Here, ΔE is the increase in potential energy, U is the energy density, A is the area of each plate, d is the distance between the plates and V is the potential difference across the plates.

The work done is equal to the increase in potential energy. Therefore,

$$F\Delta x = UA\Delta x$$

$$\Rightarrow F = UA$$

$$\Rightarrow F = \left(\frac{1}{2}\varepsilon_0 E^2\right) A$$

Electric intensity is,

$$E = \frac{V}{d}$$

Therefore, the force is,

$$F = \frac{1}{2}\varepsilon_0 \left(\frac{V}{d}\right) EA = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d} V\right) E$$

The capacitance is,

$$C = \frac{\varepsilon_0 A}{d}$$

Therefore, the force is,

$$F = \frac{1}{2}\varepsilon_0 \left(\frac{V}{d}\right) EA = \frac{1}{2}(CV)E$$

The charge stored in the capacitor is,

Therefore, the force is,

$$Q = CV$$

Therefore, the force is,

$$F = \frac{1}{2}QE$$

Therefore, the force on each plates of the parallel plate capacitor has a magnitude of $\frac{1}{2}QE$. Just outside the conductor the field is E and inside the conductor, the field is zero. Thus, the average of the fields, $\frac{E}{2}$, contributes the force. This is the origin of the factor $\frac{1}{2}$ in the force equation.

29. A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports (Fig. 2.34). Show that the capacitance of a spherical capacitor is given by $C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$where r_1 and r_2 are the radii of outer and inner spheres, respectively.

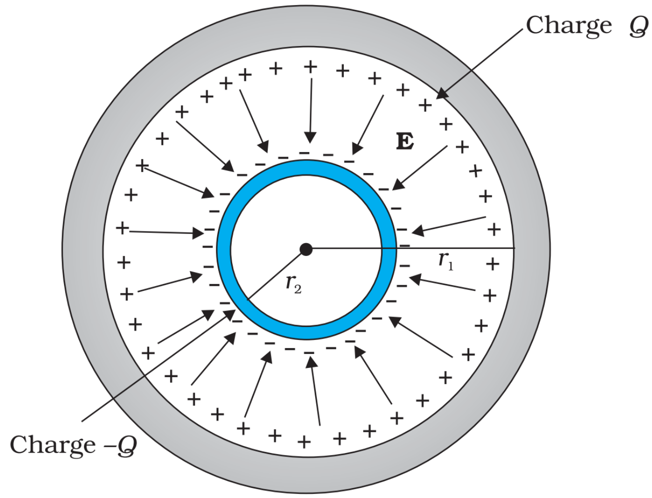


FIGURE 2.36

Solution:

Given

The radius of the outer shell is r_1 and the radius of the inner shell is r_2.

We know that the charge on the inner surface of the outer shell is $+Q$ and the charge on the outer surface of the inner shell is $-Q$.

The potential difference between the two shells is,

$$V = \frac{Q}{4\pi\varepsilon_0 r_2} - \frac{Q}{4\pi\varepsilon_0 r_1}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$= \frac{Q(r_1 - r_2)}{4\pi\varepsilon_0 r_1 r_2}$$

The capacitance of the system is,

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q(r_1 - r_2)}{4\pi\varepsilon_0 r_1 r_2}} = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$$

Hence proved.

30. A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed, and the inner sphere is given a charge of 2.5 µC. The space between the concentric spheres is filled with a liquid of dielectric constant 32.

(A) Determine the capacitance of the capacitor.

(B) What is the potential of the inner sphere?

(C) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.

Solution:

Given

The radius of the inner sphere is $r_2 = 12$ cm $= 0.12$ m.

The radius of the outer sphere is $r_1 = 13$ cm $= 0.13$ m.

The inner surface has a charge of $q = 2.5$ μC $= 2.5 \times 10^{-6}$ C.

The liquid has a dielectric constant of $\varepsilon_r = 32$.

(A) The capacitance of the capacitor is,

$$C = \frac{4\pi\varepsilon_0\varepsilon_r r_1 r_2}{r_1 - r_2}$$

$$= \frac{32\times0.12\times0.13}{9\times10^9(0.13-0.12)} = 5.5 \times 10^{-9} \text{ F}$$

Thus, the capacitance of the capacitor is 5.5×10^{-9} F.

(B) The potential of the inner sphere is,

$$V = \frac{q}{C} = \frac{2.5 \times 10^{-6}}{5.5 \times 10^{-9}} = 4.5 \times 10^2 \text{ V}$$

Thus, the potential of the inner sphere is 4.5×10^2 V.

(C) Given

The radius of the isolated sphere $r = 12$ cm $= 0.12$ m.

Capacitance of the sphere is,

$C' = 4\pi\varepsilon_0 r$

$= 4\pi \times 8.854 \times 10^{-12} \times 0.12$

$= 1.33 \times 10^{-11}$ F

Thus, the capacitance of the isolated sphere is less compared to that of the concentric spheres. The reason for this is that the outer sphere of the concentric spheres is earthed and thus, the potential is less, and the capacitance is more than the isolated sphere.

31. Answer carefully:

(A) Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by $Q_1 Q_2/4\pi\varepsilon_0 r^2$, where r is the distance between their centers?

(B) If Coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss's law be still true?

(C) A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the field line passing through that point?

(D) What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?

(E) We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there?

(F) What meaning would you give to the capacitance of a single conductor?

(G) Guess a possible reason why water has a much greater dielectric constant $(= 80)$ than say, mica $(= 6)$.

Solution:

(A) No, the magnitude of electrostatic force between the spheres is not exactly given by the relation $F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$ as there is a non-uniform charge distribution on the spheres.

(B) No, if the dependence on *r* in Coulomb's law changed to involved $1/r^3$ instead of $1/r^2$, Gauss's law will not be true.

(C) Yes. A small test charge released at rest at a point in an electrostatic field configuration will travel along the lines passing through that point if the field lines are straight. The reason for this is that the field lines give the direction of acceleration. Not of velocity.

(D) When a nucleus completes as circular or elliptical orbit, the displacement becomes zero. Thus, the work done by the field of a nucleus when it completes an orbit is zero.

(E) No, even though the electric field is discontinuous across the surface of a charged conductor, the electric potential is not. Electric potential is continuous.

(F) The capacitance of a single capacitor is considered as a parallel plate capacitor with one plate of the capacitor at infinity.

(G) Water has a greater dielectric constant compared to mica because water has unsymmetrical space compared to mica and thus, has a permanent dipole moment.

32. A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed, and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e., bending of field lines at the ends).

Solution:

Given

The length of the co-axial cylinder, $l = 15 \text{ cm} = 0.15 \text{ m}$.

Outer radius of the cylinder, $r_1 = 1.5 \text{ cm} = 0.015 \text{ m}$.

Inner radius of the cylinder, $r_2 = 1.4 \text{ cm} = 0.014 \text{ m}$.

The charge given to inner cylinder is, $q = 3.5\ \mu\text{C} = 3.5 \times 10^{-6}\ \text{C}$.

Capacitance of a co-axial coil is,

$$C = \frac{2\pi\varepsilon_0 l}{\log_e \frac{r_1}{r_2}}$$

$$= \frac{2\pi \times 8.854 \times 10^{-12} \times 0.15}{2.3026 \times \log_{10} \frac{0.15}{0.14}}$$

$$= \frac{2\pi \times 8.854 \times 10^{-12} \times 0.15}{2.3026 \times 0.0299}$$

$$= 1.2 \times 10^{-10}\ \text{F}$$

Potential difference of the inner cylinder is,

$$V = \frac{q}{C} = \frac{3.5 \times 10^{-6}}{1.2 \times 10^{-10}} = 2.92 \times 10^4\ \text{V}$$

33. A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about 107 V m^{-1}. (Dielectric strength is the maximum electric field a material can tolerate without breakdown, i.e., without starting to conduct electricity through partial ionization.) For safety, we should like the field never to exceed, say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 pF?

Solution:

The potential rating of the parallel plate capacitor is,

$V = 1\ \text{kV} = 1000\ \text{V}$

Dielectric constant of the material of the capacitor is, $\varepsilon = 3$

Dielectric strength is, $10^7\ \text{V/m}$

The field intensity should never exceed 10% of the strength of dielectric for safety. Therefore, the electric field intensity is,

$E' = 10\%E$

$= (10\%)\left(\frac{1}{100\%}\right)(10^7 \text{ V/m})$

$= 10^6 \text{ V/m}$

Capacitance of the parallel plate capacitor is,

$C = 50 \text{ pF} = 50 \times 10^{-12} \text{ F}$

The distance between the plates is,

$$d = \frac{V}{E'} = \frac{1000 \text{ V}}{10^6 \text{ V/m}} = 10^{-3} \text{ m}$$

The expression for capacitance is,

$$C = \frac{\varepsilon_0 \varepsilon A}{d}$$

Here, A is the area of each plate and ε_0 is the permittivity of free space.

Therefore, area of each plate is,

$$A = \frac{Cd}{\varepsilon_0 \varepsilon} = \frac{50 \times 10^{-12} \text{ F} \times 10^{-3} \text{ m}}{8.854 \times 10^{-12} \text{ F/m} \times 3}$$

$= 19 \times 10^{-6} \text{ m}^2$

$= (19 \times 10^{-4} \text{ m}^2)\left(\frac{1 \text{ cm}^2}{10^{-4} \text{ m}^2}\right)$

$= 19 \text{ cm}^2$

Thus, the area requires for each plates of the parallel plate capacitor to have a capacitance of 50 pF is 19 cm^2.

34. Describe schematically the equipotential surfaces corresponding to

(A) a constant electric field in the z-direction,

(B) a field that uniformly increases in magnitude but remains in a constant (say, z) direction,

(C) a single positive charge at the origin, and

(D) a uniform grid consisting of long equally spaced parallel charged wires in a plane.

Solution:

(A) The equipotential surfaces corresponding to a constant electric field in the z-direction are the equidistance planes parallel to x-y plane.

(B) For a field that uniformly increases in magnitude but remains in a constant (say, z) direction, the equipotential surfaces are the planes that are parallel to x-y plane. But when the planes get closer, the field increases.

(C) For a single positive charge at the origin, concentric spheres centered at the origin are the equipotential surfaces.

(D) For a uniform grid consisting of long equally spaced parallel charged wires in a plane. A periodically varying shape near to the grid is the equipotential surface. Gradually, these planes will reach a shape of a plane parallel to the grid at larger distances.

35. A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.

Solution:

Let the small sphere be A and the shell be B.

Potential on the inner surface of the shell is,

Potential at A = Potential due to own self + Potential due to sphere B

$$V_A = V_{AA} + V_{AB}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_A}{r_A} + \frac{1}{4\pi\varepsilon_0}\frac{q_B}{r_B}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$$

Similarly, the potential at the outer shell B is,

$$V_B = V_{BB} + V_{BA}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_B}{r_B} + \frac{1}{4\pi\varepsilon_0}\frac{q_A}{r_A}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_2}{r_2} + \frac{q_1}{r_1}\right]$$

Now,

$$V_A - V_B = \frac{q_1}{4\pi\varepsilon_0}\left[\frac{r_2 - r_1}{r_1 r_2}\right]$$

As the radius $r_2 > r_1$, the potential $V_A > V_B$.

Thus, the charge will flow from the inner sphere A to the outer sphere B when they are connected. This will continue to happen until they both attain the same potential.

Thus, the charge q_1 given to sphere A will flow to sphere B irrespective of the charge given to shell B.

36. Answer the following:

(A) The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about $100\ \mathrm{V\ m^{-1}}$. Why then do we not get an electric shock as we step out of our house into the open? (Assume the house to be a steel cage so there is no field inside!)

(B) A man fixes outside his house one evening a two meter high insulating slab carrying on its top a large aluminium sheet of area $1\ \mathrm{m^2}$. Will he get an electric shock if he touches the metal sheet next morning?

(C) The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?

(D) What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning?

Hint:

The earth has an electric field of about $100\ \mathrm{V\ m^{-1}}$ at its surface in the downward direction, corresponding to a surface charge density $= -10^{-9}\ \mathrm{C\ m^{-2}}$. Due to the slight conductivity of the atmosphere up to about 50 km (beyond which it is good conductor), about $+1800$ C is pumped every second into the earth as a whole. The earth, however, does not get discharged since thunderstorms and lightning occurring continually all over the globe pump an equal amount of negative charge on the earth.

Solution:

(A) The equipotential surfaces of open air changes and thus, keeps our body and the ground at the same potential. Thus, we do not get electric shock as we step out of our house.

(B) Yes, the man will get electric shock. This is because the aluminium sheet will get charged because of the steady discharging current in the atmosphere. The voltage will rise gradually and the rise in voltage will depend on the capacitance of the capacitor formed by the aluminium slab and the ground.

(C) The atmosphere is charged because of the thunderstorms and lightning. Thus, the atmosphere is not discharged completely even though the discharging current is about 1800 A.

(D) During thunderstorm and lightning, light energy, heat energy and sound energy are dissipated in the atmosphere.