MOVING CHARGES AND MAGNETISM

CBSE NCERT Solutions for Class 12 Physics Chapter 4

Back of Chapter Questions

4.1. A circular coil of wire consisting of 100 turns, each of radius 8.0 cm carries a current of 0.40 A. What is the magnitude of the magnetic field B at the centre of the coil?

Solution:

Given that number of turns on the circular coil, n = 100

Radius of each turn, r = 8.0 cm = 0.08 m

Current flowing in the coil, I = 0.4 A

The magnitude of the magnetic field at the centre of the coil is given by the relation,

$$B = \frac{\mu_0 nI}{2r}$$

Where, μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ T m A^{-1}

$$B = \frac{4\pi \times 10^{-7} \times 100 \times 0.4}{2 \times 0.08} = 3.14 \times 10^{-4}\ \text{T}$$

Hence, the magnitude of the magnetic field at the centre of the coil is 3.14×10^{-4} T

4.2. A long straight wire carries a current of 35 A. What is the magnitude of the field B at a point 20 cm from the wire?

Solution:

Given that Current in the wire, I = 35 A

Distance of a point from the wire, r = 20 cm = 0.2 m

The magnitude of the magnetic field at this point is given as:

$$\text{B} = \frac{\mu_0 I}{2\pi r}$$

Where, μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ T m A^{-1}

$$\text{B} = \frac{4\pi \times 10^{-7} \times 2 \times 35}{4\pi \times 0.2} = 3.5 \times 10^{-5}\ \text{T}$$

Hence, the magnitude of the magnetic field at a point of 20 cm from the wire is 3.5×10^{-5} T

4.3. A long straight wire in the horizontal plane carries a current of 50 A in the north to south direction. Give the magnitude and direction of B at a point 2.5 m east of the wire.

Solution:

Given that,

Point is 2.5 m away from the East of the wire.

The distance of the point from the wire, r = 2.5 m.

Current in the wire is 50 A

The magnitude of the magnetic field at this point is given as:

$$\text{IBI} = \frac{\mu_0\, 2I}{4\pi r}$$

Where, $\mu_0 =$ Permeability of free space $= 4\pi \times 10^{-7}$ T m A^{-1}

$$B = \frac{4\pi \times 10^{-7} \times 2 \times 50}{4\pi \times 2.5} = 4 \times 10^{-6}\ \text{T}$$

Given that point is at a distance of 2.5 m normal to the wire length and the direction of the current in the wire is vertically downward.

Hence, according to Maxwell's right-hand thumb rule, the direction of the magnetic field at the given point is vertically upward.

4.4. A horizontal overhead power line carries a current of 90 A in the east to west direction. What is the magnitude and direction of the magnetic field due to the current 1.5 m below the line?

Solution:

Given that current in the power line, I = 90 A

Location of the point below the power line, r = 1.5 m

Hence, the magnetic field at a point is given by the relation,

$$IBI = \frac{\mu_0\, 2I}{4\pi r}$$

$Where, \mu_0 = Permeability\ of\ free\ space = 4\pi \times 10^{-7}$ T m A^{-1}

$$B = \frac{4\pi \times 10^{-7} \times 2 \times 90}{4\pi \times 1.5} = 1.2 \times 10^{-5}\ \text{T}$$

The current is flowing from East to West. The point is below the power line. Hence, according to Maxwell's right-hand thumb rule, the direction of the magnetic field is towards the South.

4.5. What is the magnitude of magnetic force per unit length on a wire carrying a current of 8 A and making an angle of 30° with the direction of a uniform magnetic field of 0.15 T?

Solution:

Given that current in the wire, $I = 8$ A

The magnitude of the uniform magnetic field, $B = 0.15$ T

The angle between the wire and magnetic field, $\theta = 30^{o}$

We know that magnetic force per unit length on the wire is given as:

$f = BI\sin\theta = 0.15 \times 8 \times \sin 30° = 0.6\ \mathrm{N\,m^{-1}}$

Hence, the magnetic force per unit length on the wire is $0.6\ \mathrm{N\,m^{-1}}$

4.6. A 3.0 cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T. What is the magnetic force on the wire?

Solution:

Given that,

Length of the wire, $l = 3$ cm $= 0.03$ m

Current flowing in the wire, $I = 10$ A

The magnetic field inside the solenoid, $B = 0.27$ T

The angle between the current and magnetic field, $\theta = 90°$

The magnetic force exerted on the wire is given as

$f = BIl\sin\theta$

$f = 0.27 \times 10 \times 0.03 \times \sin 90° = 8.1 \times 10^{-2}$ N

Hence, the magnetic force on the wire is 8.1×10^{-2} N

4.7. Two long and parallel straight wires A and B carrying currents of 8.0 A and 5.0 A in the same direction are separated by a distance of 4.0 cm. Estimate the force on a 10 cm section of wire A.

Solution:

Given that,

The magnitude of the current flowing in wire A, $I_A = 8.0$ A

The magnitude of the current flowing in wire B, $I_B = 5.0$ A

Distance between the two wires, $r = 4.0$ cm $= 0.04$ m

Length of a section of wire A, $L = 10$ cm $= 0.1$ m

We know that force exerted on length L due to the magnetic field is given as.

$$F = \frac{\mu_0 I_A I_B L}{2\pi r}$$

Where, μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ T m A^{-1}

$$F = \frac{4\pi \times 10^{-7} \times 8 \times 5 \times 0.1}{2\pi \times 0.04} = 2 \times 10^{-5}\ \text{N}$$

The magnitude of the force is 2×10^{-5} N.

Here the direction of the currents in the wires is the same; therefore, the force between them is attractive in nature.

4.8. A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0 A, estimate the magnitude of B inside the solenoid near its centre.

Solution:

Given that,

Length of the solenoid, l = 80 cm = 0.8 m

There are five layers of windings of 400 turns each on the solenoid, so we need to calculate a total number of turns.

Total number of turns on the solenoid, N = 5 × 400 = 2000

Diameter of the solenoid, D = 1.8 cm = 0.018 m

The current carried by the solenoid, I = 8.0 A

The magnitude of the magnetic field inside the solenoid near its centre is given by the relation,

$$IBI = \frac{\mu_0\ NI}{l}$$

Where, μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ T m A^{-1}

$$B = \frac{4\pi \times 10^{-7} \times 2000 \times 8}{0.8} = 2.5 \times 10^{-2}\ \text{T}$$

Hence, the magnitude of the magnetic field inside the solenoid near its centre is

2.5×10^{-2} T

4.9. A square coil of side 10 cm consists of 20 turns and carries a current of 12 A. The coil is suspended vertically, and the normal to the plane of the coil makes an angle of 30^o with the direction of a uniform horizontal magnetic field of magnitude 0.80 T. What is the magnitude of torque experienced by the coil.

Solution:

Given that,

Length of a side of the square coil, $l = 10$ cm $= 0.1$ m

The magnitude of the current flowing in the coil, $I = 12$ A

Number of turns on the coil, $n = 20$

Angle made by the plane of the coil with the magnetic field, $\theta = 30°$

Strength of magnetic field, $B = 0.80$ T

The magnitude of the magnetic torque experienced by the coil in the magnetic field is given by the relation,

$\tau = n\,BIA\sin\theta$

Where, A = Area of the square coil

$= .1 \times .1$

$= 0.01\ m^2$

So,

$\tau = 20 \times 0.8 \times 12 \times 0.01 \times \sin 30°$

$= 0.96$ N m

Hence, the magnitude of the torque experienced by the coil is $= 0.96$ N m

4.10. Two moving coil meters, M_1 and M_2 have the following particulars: $R_1 = 10\ \Omega$, $N_1 = 30$, $A_1 = 3.6 \times 10^{-3}\ m^2$, $B_1 = 0.25$ T $R_2 = 14\ \Omega$, $N_2 = 42$, $A_2 = 1.8 \times 10^{-3}\ m^2$, $B_2 = 0.50$ T

(The spring constants are identical for the two meters).

Determine the ratio of

(a) current sensitivity and

(b) voltage sensitivity of M_2 and M_1.

Solution:

Given that for moving coil meter M_1:

Resistance, $R_1 = 10\ \Omega$

Number of turns, $N_1 = 30$

Area of cross-section $A_1 = 3.6 \times 10^{-3}\ m^2$

Magnetic field strength, $B_1 = 0.25$ T

Spring constant $K_1 = K$

For moving coil meter M_2:

Resistance, $R_2 = 14\ \Omega$

Number of turns, $N_2 = 42$

Area of cross-section, $A_2 = 1.8 \times 10^{-3}\ m^2$

Magnetic field strength, $B_2 = 0.50\ T$

Spring constant, $K_2 = K$

(a) Current sensitivity of M_1 is given as:

$$I_{s1} = \frac{N_1 B_1 A_1}{K_1}$$

And, current sensitivity of M_2 is given as:

$$I_{s2} = \frac{N_2 B_2 A_2}{K_2}$$

$$\therefore \quad \frac{I_{s2}}{I_{s1}} = \frac{N_2 B_2 A_2}{N_1 B_1 A_1} = \frac{42 \times 0.5 \times 1.8 \times 10^{-3}}{30 \times 0.25 \times 3.6 \times 10^{-3}} = 1.4$$

Hence, the ratio of current sensitivity of M_2 to M_1 is 1.4.

(b) Voltage sensitivity for M_2 given as:

$$V_{s2} = \frac{N_2 B_2 A_2}{K_2 R_2}$$

And, voltage sensitivity of M_1 is given as:

$$V_{s1} = \frac{N_1 B_1 A_1}{K_1 R_1}$$

$$\therefore \quad \frac{V_{s2}}{V_{s1}} = \frac{N_2 B_2 A_2 K_1 R_1}{N_1 B_1 A_1 K_2 R_2} = \frac{42 \times 0.5 \times 1.8 \times 10^{-3} \times K}{K \times 14 \times 30 \times 0.25 \times 3.6 \times 10^{-3}} = 1$$

Hence, the ratio of voltage sensitivity of M_2 to M_1 is 1.

4.11. In a chamber, a uniform magnetic field of 6.5 G ($1\ G = 10^{-4}\ T$) is maintained. An electron is shot into the field with a speed of $4.8 \times 10^6\ m\ s^{-1}$ normal to the field. Explain why the path of the electron is a circle. Determine the radius of the circular orbit. ($e = 1.6 \times 10^{-19}\ C$, $m_e = 9.1 \times 10^{-31}\ kg$)

Solution:

Given that,

Magnetic field strength, $B = 6.5\ G = 6.5 \times 10^{-4} T$

Speed of the electron, $v = 4.8 \times 10^6\ m/s$

Charge on the electron, $e = 1.6 \times 10^{-19}C$

Mass of the electron, $m_e = 9.1 \times 10^{-31}$ kg

The angle between the shot electron and magnetic field, $\theta = 90°$

The magnetic force exerted on the electron in the magnetic field is given as:

$F = evB \sin\theta$

Electron is moving around a circular path. To rotate in a circular path, electron needs a centripetal force. Centripetal force is provided by the magnetic force.

Hence, the centripetal force exerted on the electron,

$$F_e = \frac{mv^2}{r}$$

For equilibrium, the centripetal force exerted on the electron is equal to the magnetic force, i.e.,

$$F_e = F$$

$$\Rightarrow \frac{mv^2}{r} = evB \sin\theta$$

$$\Rightarrow r = \frac{mv}{eB \sin\theta}$$

So,

$$r = \frac{9.1 \times 10^{-31} \times 4.8 \times 10^6}{6.5 \times 10^{-4} \times 1.6 \times 10^{-19} \times \sin 90°} = 4.2 \times 10^{-2}\text{m} = 4.2 \text{ cm}$$

Hence, the radius of the circuit orbit of the electron is 4.2 cm

4.12. In Exercise 4.11 obtain the frequency of revolution of the electron in its circular orbit. Does the answer depend on the speed of the electron? Explain.

Solution:

Given that,

Magnetic field strength, $B = 6.5 \times 10^{-4}$ T

Charge of the electron, $e = 1.6 \times 10^{-19}$ C

Mass of the electron, $m_e = 9.1 \times 10^{-31}$kg

The velocity of the electron, $v = 4.8 \times 10^6$ m/s

The radius of the orbit, $r = 4.2$ cm $= 0.042$ m

Assume that the frequency of revolution of the electron is ν

Angular frequency of the electron $= \omega = 2\pi\nu$

The relation between the velocity of the electron and angular frequency is $v = r\omega$

In the circular orbit, the centripetal force is balanced by the magnetic force of the electron.

Hence, we can write:

$$\frac{mv^2}{r} = evB$$

$$\Rightarrow eB = \frac{mv}{r} = \frac{m(r\omega)}{r} = \frac{m(r.2\pi\nu)}{r}$$

$$\Rightarrow \nu = \frac{eB}{2\pi m}$$

Here we can see that expression for frequency is independent of the speed of the electron.

$$\nu = \frac{6.5 \times 10^{-4} \times 1.6 \times 10^{-19}}{2 \times 3.14 \times 9.1 \times 10^{-31}} = 1.82 \times 10^{7}\text{Hz} \approx 18\text{MHz}$$

Hence, the frequency of the electron is around 18 MHz and is independent of the speed of the electron.

4.13. (a) A circular coil of 30 turns and radius 8.0 cm carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.0 T. The field lines make an angle of 60° with the normal of the coil. Calculate the magnitude of the counter-torque that must be applied to prevent the coil from turning.

(b) Would your answer change, if the circular coil in (a) were replaced by a planar coil of some irregular shape that encloses the same area? (All other particulars are also unaltered.)

Solution:

(a) Given that,

Number of turns in circular coil = 30

Radius of the coil, r = 8.0 cm = 0.08 m

Area of the coil= $\pi r^2 = \pi \times 0.08^2 = 0.0201 \text{ m}^2$

The magnitude of the current flowing in the coil, $I = 6.0$ A

Magnetic field strength, $B = 1$ T

The angle between the field lines and normal with the coil surface, $\theta = 60°$

In this case, the coil is experiencing a torque in the magnetic field, and due to this, it will turn.

To oppose turning of the coil, we must apply the counter torque.

Torque, $\tau = n\,BIA \sin\theta$

$\tau = 30 \times 6 \times 1 \times 0.0201 \times sin60^\circ = 3.1333$ N m

(b) From the formula of torque $\tau = n\,BIA \sin\theta$, we can say that the magnitude of the applied torque is independent on the shape of the coil. It depends on the area of the coil. Hence, Magnitude of torque would not change if the circular coil in the above case is replaced by a planar coil of some irregular shape that encloses the same area.

Additional Exercises

4.14. Two concentric circular coils X and Y of radii 16 cm and 10 cm, respectively, lie in the same vertical plane containing the north to south direction. Coil X has 20 turns and carries a current of 16 A; coil Y has 25 turns and carries a current of 18 A. The sense of the current in X is anticlockwise, and clockwise in Y, for an observer looking at the coils facing west. Give the magnitude and direction of the net magnetic field due to the coils at their centre.

Solution:

Given that,

The radius of X, $r_1 = 16$ cm $= 0.16$ m

The radius of coil Y, $r_2 = 10$ cm $= 0.1$ m

Number of turns on coil X, $N_1 = 20$

Number of turns on coil Y, $N_2 = 25$

Current in coil X, $I_1 = 16$ A

Current in coil Y, $I_2 = 18$ A

Magnetic field due to coil X at their centre is given by the expression,

$$B_1 = \frac{\mu_0 N_1 I_1}{2r_1}$$

Where, μ_0 = permeability of free space $= 4\pi \times 10^{-7}$ T m A^{-1}

$$B_1 = \frac{4\pi \times 10^{-7} \times 20 \times 16}{2 \times 0.16} = 4\pi \times 10^{-4}\text{T (Towards east)}$$

Magnetic field due to coil Y at their centre is given by the expression.

$$B_2 = \frac{\mu_0 N_2 I_2}{2r_2}$$

Where, μ_0 = permeability of free space $= 4\pi \times 10^{-7}$ T m A^{-1}

$B_1 = \frac{4\pi\times10^{-7}\times25\times18}{2\times0.10} = 9\pi \times 10^{-4}$ T (Towards west)

Hence, the net magnetic field can be obtained as the difference of magnitude of the magnetic field in coil X and coil Y

$B = B_2 - B_1 = 9\pi \times 10^{-4}\text{T} - 4\pi \times 10^{-4}$ T

$= 5\pi \times 10^{-4}$ T

$= 5 \times 3.14 \times 10^{-4}$ T $= 1.57 \times 10^{-3}$ T (Towards West)

4.15. A magnetic field of 100 G (1 G = 10^{-4} T) is required which is uniform in a region of linear dimension about 10 cm and area of cross-section about 10^{-3} m^2. The maximum current-carrying capacity of a given coil of wire is 15 A, and the number of turns per unit length that can be wound round a core is at most 1000 turns m^{-1}. Suggest some appropriate design particulars of a solenoid for the required purpose. Assume the core is not ferromagnetic.

Solution:

Given that,

Strength of magnetic field, $B = 100\text{ G} = 100 \times 10^{-4}$ T

Number of turns per unit length, $n = 1000$ turns m^{-1}

Current flowing in the coil, $I = 15$ A

Permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}

Magnetic field is given by the relation,

$B = \mu_0 nI$

$$\Rightarrow nI = \frac{B}{\mu_0} = \frac{100 \times 10^{-4}}{4\pi \times 10^{-7}} = 7957.74 \approx 8000\text{ A m}^{-1}$$

If the length of the coil is taken as 50 cm, radius 4 cm, number of turns 400, and current 10 A, then these values are not unique for the given purpose. There is always a possibility of some changes with limits.

4.16. For a circular coil of radius R and N turns carrying current I; the magnitude of the magnetic field at a point on its axis at a distance x from its centre is given by,

$$B = \frac{\mu_0 IR^2N}{2(X^2 + R^2)^{\frac{3}{2}}}$$

(a) Show that this reduces to the familiar result for the field at the centre of the coil.

(b) Consider two parallel co-axial circular coils of equal radius R, and number of turns N, carrying equal currents in the same direction, and separated by a distance R. Show that the field on the axis around the mid-point between the coils is uniform over a distance that is small as compared to R, and is given by,

$$B = 0.72 \frac{\mu_0 NI}{R}$$

Solution:

Given that,

The radius of circular coil = R

Number of turns on the coil = N

Current in the coil = I

The magnetic field at a point on its axis at distance x is given by the relation,

$$B = \frac{\mu_0 I R^2 N}{2(x^2 + R^2)^{\frac{3}{2}}}$$

Where, μ_0 = permeability of free space = $4\pi \times 10^{-7}$ T m A^{-1}

(a) If the magnetic field at the centre of the coil is considered, the $x = 0$.

$$\therefore B = \frac{\mu_0 I R^2 N}{2R^3} = \frac{\mu_0 NI}{2R}$$

This is the expression for the magnetic field at the centre of the coil.

(b) Assume radii of two parallel co-axial circular coils is R

Assume the number of turns on each coil = N

Current in both coils = I

Distance between both the coils = R

Let us consider point Q at distance d from the centre.

Then, one coil is at a distance of $\frac{R}{2} + d$ from point Q.

The magnetic field at point Q is given as. $\therefore B_1 = \frac{\mu_0 NIR}{2\left[\left(\frac{R}{2}+d\right)^2 + R^2\right]^{\frac{3}{2}}}$

Also, the other coil is at a distance of $\frac{R}{2} - d$ from point Q.

Magnetic field due to this coil is given as: $\therefore B_2 = \frac{\mu_0 NIR^2}{2\left[\left(\frac{R}{2}-d\right)^2 + R^2\right]^{\frac{3}{2}}}$

Total magnetic field

$B = B_1 + B_2$

$$=\frac{\mu_0 IR^2}{2}\left[\left\{\left(\frac{R}{2}-d\right)^2+R^2\right\}+\left\{\left(\frac{R}{2}+d\right)^2+R^2\right\}^{\frac{3}{2}}\right]$$

$$=\frac{\mu_0 IR^2}{2}\left[\left\{\left(\frac{5R}{4}-d^2-Rd\right)^{\frac{3}{2}}+\right\}+\left\{\left(\frac{5R^2}{4}+d^2+Rd\right)\right\}^{\frac{3}{2}}\right]$$

$$=\frac{\mu_0 IR^2}{2}\left(\frac{5R^2}{4}\right)^{\frac{3}{2}}\left[\left(1+\frac{4}{5}\frac{d^2}{R^2}-\frac{4}{5}\frac{d}{R}\right)^{\frac{3}{2}}+\left(1+\frac{4}{5}\frac{d^2}{R^2}+\frac{4}{5}\frac{d}{R}\right)^{\frac{3}{2}}\right]$$

For $d << R$, neglecting the factor $\frac{d^2}{R^2}$,

We get:

$$\approx\frac{\mu_0 IR^2}{2}\times\left(\frac{5R^2}{4}\right)^{\frac{3}{2}}\times\left[\left(1-\frac{4d}{5R}\right)^{\frac{3}{2}}+\left(1+\frac{4d}{5R}\right)^{\frac{3}{2}}\right]$$

$$\approx\frac{\mu_0 IR^2 N}{2R^3}\times\left(\frac{4}{5}\right)^{\frac{3}{2}}\left[1-\frac{6d}{5R}+1+\frac{6d}{5R}\right]$$

$$B=\left(\frac{4}{5}\right)^{\frac{3}{2}}\frac{\mu_0 IN}{R}=0.72\left(\frac{\mu_0 IN}{R}\right)$$

Hence, it is proved that the field on the axis around the mid-[point between the coils is uniform

4.17. A toroid has a core (non-ferromagnetic) of inner radius 25 cm and outer radius 26 cm, around which 3500 turns of a wire are wound. If the current in the wire is 11 A, what is the magnetic field

(a) outside the toroid

(b) inside the core of the toroid, and

(c) in the empty space surrounded by the toroid?

Solution:

Given that,

The inner radius of the toroid, $r_i = 25\text{ cm} = 0.25\text{ m}$

The outer radius of the toroid, $r_o = 26\text{ cm} = 0.26\text{ m}$

Number of turns on the coil, N = 3500

Current in the coil, I = 11 A

(a) We know that Magnetic field outside a toroid is zero but for inside the core of a toroid magnetic field will be nonzero.

(b) The magnetic field inside the core of a toroid is given by the expression,

$$IBI = \frac{\mu_0 NI}{l}$$

$l = length\ of\ toroid$

$$= 2\pi \frac{r_i + r_o}{2}$$

$$= \pi(0.26 + 0.25) = 0.51\pi$$

$$B = \frac{4\pi \times 10^{-7} \times 3500 \times 11}{0.51\pi} = 3 \times 10^{-2}\ T$$

Hence inside the core of toroid magnetic field is 3×10^{-2} T

4.18. Answer the following questions:

(a) A magnetic field that varies in magnitude from point to point but has a constant direction (east to west) is set up in a chamber. A charged particle enters the chamber and travels undeflected along a straight path with constant speed. What can you say about the initial velocity of the particle?

(b) A charged particle enters an environment of a strong and non-uniform magnetic field varying from point to point both in magnitude and direction and comes out of it following a complicated trajectory. Would its final speed equal the initial speed if it suffered no collisions with the environment?

(c) An electron is moving west to east enters a chamber having a uniform electrostatic field in the north to south direction. Specify the direction in which a uniform magnetic field should be set up to prevent the electron from deflecting from its straight-line path.

Solution:

(a) in the given question, the magnetic field is in the constant direction from East to West.

According to the question, a charged particle travels undeflected along a straight path with constant speed. It is only possible if the magnetic force experienced by the charged particle is zero.

The magnitude of the magnetic force on a moving charged particle in a magnetic field is given by $F = q v B \sin\theta$. (where θ is the angle between v and B).

Here Force will be zero if $\sin\theta = 0$ $(as\ v \neq 0, q \neq 0, B \neq 0)$. This shows that the angle between the velocity and magnetic field is 0° or 180°.

Thus, the charged particle moves parallel or anti-parallel to the magnetic field B.

(b) Yes, we can say that the final speed of the charged particle will be equal to its initial speed. This is because magnetic force can change the direction of velocity, but not its magnitude.

(c) As the direction of the electric field is from North to South, that means the plate in North is positive and in the South is negative. Thus, the electrons (negatively charged) attract towards the positive plate that means it moves towards North. If we want that there should be no deflection in the path of an electron, then the magnetic force should be in South direction. This moving electron will be undeflected if the electric force acting on it is equal and opposite of magnetic field. Magnetic force is directed towards the South. According to Fleming's left-hand rule, the magnetic field should be applied in a vertically downward direction.

4.19. An electron emitted by a heated cathode and accelerated through a potential difference of 2.0 kV, enters a region with the uniform magnetic field of 0.15 T. Determine the trajectory of the electron if the field

(a) is transverse to its initial velocity

(b) makes an angle of 30° with the initial velocity.

Solution:

Given that,

Magnetic field strength, $\mathrm{B} = 0.15\ \mathrm{T}$

Charge on the electron, $\mathrm{e} = 1.6 \times 10^{-19}\ \mathrm{C}$

Mass of the electron, $\mathrm{m} = 9.1 \times 10^{-31}\ \mathrm{kg}$

The potential difference, $V = 2.0\ \mathrm{kV} = 2 \times 10^{3}\ \mathrm{V}$

We know that the kinetic energy of the electron $=$ eV

$$\Rightarrow \mathrm{eV} = \frac{1}{2}\mathrm{m}v^2$$

$$v = \sqrt{\frac{2\mathrm{eV}}{\mathrm{m}}} \ldots.(1)$$

Where v = velocity of the electron

(a) Here electron is doing circular motion, and for circular motion, we need centripetal force. Magnetic force on the electron provides the required centripetal force of the electron.

Assume that electron is moving in a circular path of radius r.

Magnetic force on the electron $= \mathrm{B\,e\,v}$

Centripetal force = $\frac{mv^2}{r}$

$\therefore$ B e $v = \dfrac{\mathrm{m}v^2}{\mathrm{r}}$

$r = \frac{mv}{Be}$ …… (2)

From equations (1) and (2), we get

$$r = \frac{m}{Be}\left[\frac{2eV}{m}\right]^{\frac{1}{2}}$$

$$= \frac{9.1 \times 10^{-31}}{0.15 \times 1.6 \times 10^{-19}} \times \left(\frac{2 \times 1.6 \times 10^{-19} \times 2 \times 10^{3}}{9.1 \times 10^{-31}}\right)^{\frac{1}{2}}$$

$= 1.01 \times 10^{-3}$ m

$= 1$ mm

Hence, the electron has a circular trajectory of radius 1.0 mm normal to the magnetic field.

(b) When the magnetic field makes an angle θ of 30° with an initial velocity, the initial velocity will be,

$v_1 = v \sin\theta$

From equation (2), we can write the expression for a new radius as:

$$r_1 = \frac{mv_1}{Be}$$

$$= \frac{m\ v \sin\theta}{B\ e}$$

$$= \frac{9.1 \times 10^{-11}}{0.15 \times 1.6 \times 10^{-19}}\left[\frac{2 \times 1.6 \times 10^{-19} \times 2 \times 10^{3}}{9 \times 10^{-31}}\right]^{\frac{1}{2}} \times \sin 30°$$

$= 0.5 \times 10^{-3}$ m

$= 0.5$ mm

Hence, the electron will move in a helical path of radius 0.5 mm along the magnetic field direction.

4.20. A magnetic field set up using Helmholtz coils (described in Exercise 4.16) is uniform in a small region and has a magnitude of 0.75 T. In the same region, a uniform electrostatic field is maintained in a direction normal to the common axis of the coils. A narrow beam of single species) charged particles all accelerated through 15 kV enters this region in a direction perpendicular to both the axis of the coils and the electrostatic field. If the beam remains undeflected when the electrostatic field is $9 \times 10^{-5}\ \mathrm{V\,m^{-1}}$, make a simple guess as to what the beam contains. Why is the answer not unique?

Solution:

Given that,

A magnetic field, $B = 0.75\ \mathrm{T}$

Accelerating voltage, $V = 15\ \mathrm{kV} = 15 \times 10^3\ \mathrm{V}$

Electrostatic field, $E = 9 \times 10^{-5}\ \mathrm{V\,m^{-1}}$

Mass of the electron = m,

Charge of the electron = e,

The velocity of the electron = v

The kinetic energy of the electron $=$ e V

$$\Rightarrow \frac{1}{2} \mathrm{m}\, v^2 = \mathrm{e\,V}$$

$$\therefore \frac{\mathrm{e}}{\mathrm{m}} = \frac{v^2}{2\mathrm{V}} \ldots\ldots (1)$$

It is given in the question that the particle remains un-deflected by electric and magnetic fields, we can say that the electric field is balanced by the magnetic field.

$$\therefore \mathrm{e\,E} = \mathrm{e}\, v\, \mathrm{B}$$

$$\mathrm{v} = \frac{\mathrm{E}}{\mathrm{B}} \ldots\ldots\ldots (2)$$

Put the value of v from equation (2) in equation (1), we get

$$\frac{\mathrm{e}}{\mathrm{m}} = \frac{1}{2} \frac{\left(\frac{\mathrm{E}}{\mathrm{B}}\right)}{\mathrm{V}} = \frac{\mathrm{E}^2}{2\mathrm{VB}^2}$$

$$= \frac{(9.0 \times 10^{-5})^2}{2 \times 15000 \times (0.75)^2} = 4.8 \times 10^{-13}\ \mathrm{C/kg}$$

This value of specific charge e/m is equal to the value of deuteron or deuterium ions. This is not a unique answer. Other possible answers are He^{++}, Li^{++}.etc.

4.21. A straight horizontal conducting rod of length 0.45 m and mass 60 g is suspended by two vertical wires at its ends. A current of 5.0 A is set up in the rod through the wires.

(a) What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero?

(b) What will be the total tension in the wires if the direction of current is reversed, keeping the magnetic field the same as before?

(Ignore the mass of the wires.) $g = 9.8\ \text{m s}^{-2}$.

Solution:

Given that length of the rod, $l = 0.45$ m

The magnitude of mass suspended by the wires, $m = 60\ \text{g} = 60 \times 10^{-3}$ kg

Acceleration due to gravity, $g = 9.8\ \text{m s}^{-2}$.

The magnitude of the current in the rod flowing through the wire, I = 5 A

(a) tension in the wire will be zero if the magnetic field (B) is equal and opposite to the weight of the wire

i.e., $B\,I\,l = mg$

$$B = \frac{mg}{Il}$$

$$B = \frac{60 \times 9.8 \times 10^{-3}}{5 \times 0.45} = 0.26\ \text{T}$$

A horizontal magnetic field of 0.26 T, which is normal to the length of the conductor is required to get zero tension in the wire.

According to Fleming's left-hand rule, the direction of magnetic force will be in an upward direction.

(b) Now If the direction of the current is reversed, then the force due to the magnetic field and the weight of the wire act in a vertically downward direction.

Total tension in the wire

$= BIl + mg = 0.26 \times 5 \times 0.45 + (60 \times 10^{-3}) \times 9.8 = 1.176$ N

4.22. The wires which connect the battery of an automobile to its starting motor carry a current of 300 A (for a short time). What is the force per unit length between the wires if they are 70 cm long and 1.5 cm apart? Is the force attractive or repulsive?

Solution:

Given that,

Current in both wires, $I = 300$ A

Distance between the wires, $r = 1.5\ \text{cm} = 0.015$ m

Length of the two wires, $l = 70\ \text{cm} = 0.7$ m

Force per unit length between the two wires is given by the relation,

$$F = \frac{\mu_0 I^2}{2\pi r}$$

$$F = \frac{4\pi \times 10^{-7} \times 300 \times 300}{2\pi \times 0.015} = 1.2\ \text{N/m}$$

Since the direction of the current in the wires is opposite so repulsive force will be generated between them.

4.23. A uniform magnetic field of 1.5 T exists in a cylindrical region of radius 10.0 cm, its direction parallel to the axis along east to west. A wire carrying current of 7.0 A in the north to south direction passes through this region. What is the magnitude and direction of the force on the wire if,

(a) the wire intersects the axis,

(b) the wire is turned from N-S to northeast-northwest direction,

(c) the wire in the N-S direction is lowered from the axis by a distance of 6.0 cm?

Solution:

Given that,

Strength of the magnetic field is, $B = 1.5\ \text{T}$

Radius of the cylindrical region, $r = 10\ \text{cm} = 0.1\ \text{m}$

Current in the wire passing through the cylindrical region, $I = 7\ \text{A}$

(a) Here the wire is intersecting the axis, so the length of the wire is the diameter of the cylindrical region. Thus, $l = 2r = 0.2\ \text{m}$

The angle between the magnetic field and current, $\theta = 90°$

We know that expression for magnetic force acting on the wire is,

$F = BIl \sin\theta = 1.5 \times 7 \times 0.2 \times \sin 90° = 2.1\ \text{N}$

Hence, a force of magnitude 2.1 N acts on the wire in a vertically downward direction.

(b) The new length of the wire after turning it to the Northeast-Northwest direction can be written as:

$$l_1 = \frac{l}{\sin\theta}$$

Now angle between the magnetic field and current, $\theta = 45°$

Force on the wire,

$= BIl_1 \sin\theta$

$= 1.5 \times 7 \times 0.2$

$= 2.1\ \text{N}$

Hence, a force of 2.1 N acts vertically downward on the wire and force is independent of angle θ because l sinθ is fixed.

(c) The wire is lowered from the axis by distance, d = 6.0 cm

Assume that l_2 be the new length of the wire.

$$\therefore \left(\frac{l_2}{2}\right)^2 = 4(\mathrm{d} + \mathrm{r})$$

$$= 4(10 + 6) = 4(16)$$

$$\therefore l_2 = 8 \times 2 = 16 \text{ cm} = 0.16 \text{ m}$$

The magnetic force exerted on the wire,

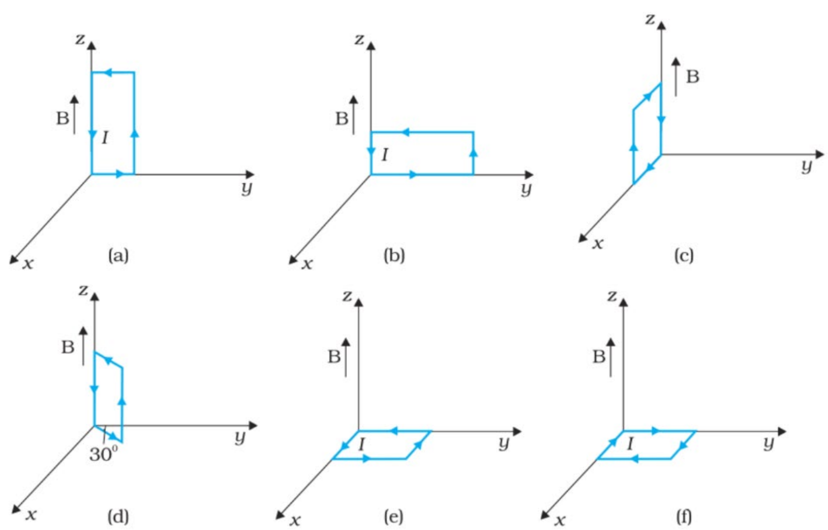
$$\mathrm{f}_2 = \mathrm{BI}l_2$$

$$= 1.5 \times 7 \times 0.16$$

$$= 1.68 \text{ N}$$

Hence, a force of magnitude 1.68 N acts a vertically downward direction on the wire.

4.24. A uniform magnetic field of 3000 G is established along the positive z-direction. A rectangular loop of sides 10 cm and 5 cm carries a current of 12 A. What is the torque on the loop in the different cases shown in Fig. 4.28? What is the force on each case? Which case corresponds to a stable equilibrium?



Solution:

Given that magnetic field strength, $\mathrm{B} = 3000 \text{ G} = 3000 \times 10^{-4}\text{T} = 0.3 \text{ T}$

Length of the rectangular loop, $l = 10$ cm

Width of the rectangular loop, $b = 5$ cm

Area of the loop, $A = 1 \times b \times 5 = 50\ \text{cm}^2 = 50 \times 10^{-4}\ \text{m}^2$

Current in the loop, $1 = 12$ A

Assume that the anti-clockwise direction of the current is positive and vice-versa:

(a) We know that expression of torque, $\vec{\tau} = I\,\overrightarrow{A} \times \vec{B}$

From the given figure, we can say that A is normal to the y-z plane and B is directed along the z-axis.

$\vec{B} = 0.3\,\hat{k}, \overrightarrow{A} = 50 \times 10^{-4}\hat{\imath}$

$\therefore \tau = 12 \times (50 \times 10^{-4})\hat{\imath} \times 0.3\,\hat{k}$

$= -1.8 \times 10^{-2}\,\hat{\jmath}$ Nm

The torque is 1.8×10^{-2} Nm along the negative y-direction and angle between A and B is zero; hence, the force on the loop will be zero.

(b) In this case, the direction of the current is the same as in case (a)

Hence answer is the same as a case (a).

(c) Expression of torque is $\tau = I\,\vec{A} \times \vec{B}$

From the given figure, we can say that area vector A is normal to the x-z plane and B is directed along the z-axis.

$\therefore \tau = 12 \times (50 \times 10^{-4})\,\hat{\jmath} \times 0.3\,\hat{k}$

$= -1.8 \times 10^{-2}\,\hat{\imath}$ Nm

The torque is 1.8×10^{-2} Nm along the negative x-direction and the force is zero. on Magnitude of torque is given as:

(d) Torque $|\tau| = I \times A \times B$

$= 12 \times 50 \times 10^{-4} \times 0.3$

$= 1.8 \times 10^{-2}$ Nm

The torque is1.8×10^{-2} Nm at an angle of $240°$ with positive *x*-direction. The force is zero.

(e) Torque $\tau = I\vec{A} \times \vec{B}$

$= (50 \times 10^{-4} \times 12)\ \hat{k} \times 0.3\,\hat{k}$

$= 0$

Hence, the torque is zero. The force is also zero.

(f) Expression of torque $\tau = I\vec{A} \times \vec{B}$

$= (50 \times 10^{-4} \times 12)\ \hat{k} \times 0.3\,\hat{k}$

$= 0$

Hence, the torque is zero. The force is also zero.

In case (e), the direction of $I\vec{A}$ and $\vec{B}$ is the same, and the angle between them is zero. If displaced, they come back to an equilibrium. Hence, it is in a stable equilibrium.

Whereas, in case (f), the direction of $I\vec{A}$ and $\vec{B}$ is the opposite. The angle between them is 180°. If disturbed, it does not come back to its original position. Hence, its equilibrium is unstable.

4.25. A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in the coil is 5.0 A, what is the

(a) total torque on the coil,

(b) the total force on the coil,

(c) average force on each electron in the coil due to the magnetic field?

(The coil is made of copper wire of cross-sectional area 10^{-5}m^2, and

the free electron density in copper is given to be about 10^{29}m^{-3})

Solution:

Given that,

Radius of the coil, $r = 10\text{ cm} = 0.1\text{ m}$

Strength of the magnetic field, $B = 0.10\text{ T}$

Number of turns on the circular coil, n=20

Current in the coil, $I = 5.0\text{ A}$

(a) Because of the uniform magnetic field, The total torque on the coil will be zero.

(b) Because of the uniform magnetic field, The total force on the coil will be zero.

(c) The cross-sectional area of copper coil, $A = 10^{-5}\text{ m}^2$

Number of free electrons per cubic metre in copper, $N = 10^{29}/\text{m}^3$

Charge on the electron, $e = 1.6 \times 10^{-19}\text{ C}$

Magnetic force, $F = Bev_d$ where, v_d =Drift velocity of electrons

$$v_d = \frac{\mathrm{I}}{\mathrm{NeA}}$$

$$\therefore \mathrm{F} = \frac{\mathrm{Bel}}{\mathrm{NeA}}$$

$$= \frac{0.10 \times 5.0}{10^{29} \times 10^{-5}} = 5 \times 10^{-25}\ \mathrm{N}$$

Hence, the average force on each electron is 5×10^{-25} N.

4.26. A solenoid 60 cm long and of radius 4.0 cm has 3 layers of windings of 300 turns each. A 2.0 cm long wire of mass 2.5 g lies inside the solenoid (near its centre) normal to its axis; both the wire and the axis of the solenoid are in the horizontal plane. The wire is connected through two leads parallel to the axis of the solenoid to an external battery which supplies a current of 6.0 A in the wire. What value of current (with an appropriate sense of circulation) in the windings of the solenoid can support the weight of the wire? g = 9.8 m s^{-2})

Solution:

Given that ,

Radius of the solenoid, r = 4.0 cm = 0.04 m

Length of the solenoid, L = 60 cm = 0.6 m

It is given that there are 3 layers of windings of 300 turns each.

∴ Total number of turns, n = 3 × 300 = 900,

Length of the wire, I = 2 cm = 0.02 m

Mass of the wire, m = 2.5 g = 2.5×10^{-3} kg

Current flowing through the wire, i = 6 A

Acceleration due to gravity, g = 9.8 $\mathrm{m/s^2}$

The magnetic field produced inside the solenoid, B = $\frac{\mu_0 \mathrm{nl}}{\mathrm{L}}$

Where,

P= Permeability of free space = $4\pi \times 10^{-7}\mathrm{Tm\ A^{-1}}$

I = Current is flowing through the windings of the solenoid

Magnetic force is given by the relation,

$$F = Bil$$

$$= \frac{\mu_0 nI}{L} il$$

Also, the force on the wire is equal to the weight of the wire.

$$\therefore mg = \frac{\mu_0 nIil}{L}$$

$$\mathrm{I} = \frac{\mathrm{mgL}}{\mu_0 \mathrm{nil}}$$

$$= \frac{2.5 \times 10^{-3} \times 9.8 \times 0.6}{4\pi \times 10^{-7} \times 900 \times 0.02 \times 6} = 108\ \mathrm{A}$$

Hence, the current flowing through the solenoid is 108 A.

4.27. A galvanometer coil has a resistance of $12\ \Omega$ and the metre shows full-scale deflection for a current of 3 mA. How will you convert the metre into a voltmeter of range 0 to 18 V?

Solution:

Given that resistance of the galvanometer coil, $\mathrm{G} = 12\ \Omega$

Current for which there is a full-scale deflection, $I_g = 3\ \mathrm{mA} = 3 \times 10^{-3}\mathrm{A}$

The initial reading of the voltmeter is 0; we have to convert to 18 V.

$\mathrm{V} = 18\ \mathrm{V}$

We know that to convert a galvanometer into a voltmeter, we need to connect a resistor of resistance R in series with the galvanometer.

This resistance is given as $R = \frac{V}{I_g} - G$

$$= \frac{18}{3 \times 10^{-3}} - 12 = 5988\ \Omega$$

Hence, a resistor of resistance $5988\ \Omega$ is to be connected in series with the galvanometer.

4.28. A galvanometer coil has a resistance of $15\ \Omega$, and the metre shows full-scale deflection for a current of 4 mA. How will you convert the metre into an ammeter of range 0 to 6 A?

Solution:

Given that,

The resistance of the galvanometer coil, $\mathrm{G} = 15\ \Omega$

Current for which the galvanometer shows full-scale deflection,

$\mathrm{I_g} = 4\ \mathrm{mA} = 4 \times 10^{-3}\mathrm{A}$

Range of the ammeter is 0; we have to change it to 6 A.

Current, I $= 6$ A

We know that a shunt resistor of resistance S is to be connected in parallel with the galvanometer to convert it into an ammeter.

The value of S is given as $S = \frac{I_g G}{I - i_g}$

$$S = \frac{4 \times 15 \times 10^{-3}}{6 - 4 \times 10^{-3}} = 0.01\ \Omega$$

Hence, a shunt resistor of $0.01\ \Omega$ is to be connected in parallel with the galvanometer.

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