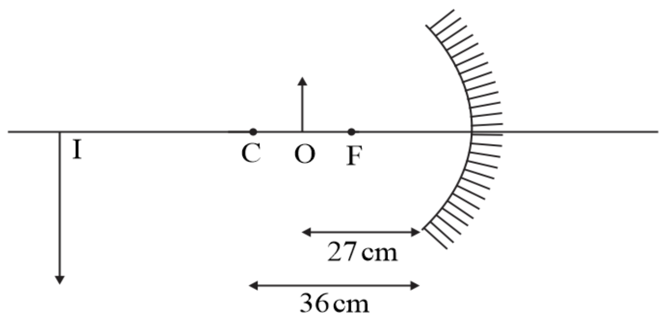
RAY OPTICS

CBSE NCERT Solutions for Class 12 Physics Chapter 9

Back of Chapter Questions

9.1. A small candle, 2.5 cm in size is placed at 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?

Solution:



Given,

The Object distance $u = -27$ cm

Focal length $f = -\frac{R}{2} = -\frac{36 \text{ cm}}{2} = -18$ cm

From the mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow v = \frac{uf}{u-f}$$

$$\Rightarrow v = \frac{(-27)(-18)}{(-27)-(-18)} = -54 \text{ cm}$$

Magnification $m = -\frac{v}{u} = -\frac{-54}{-27} = -2$

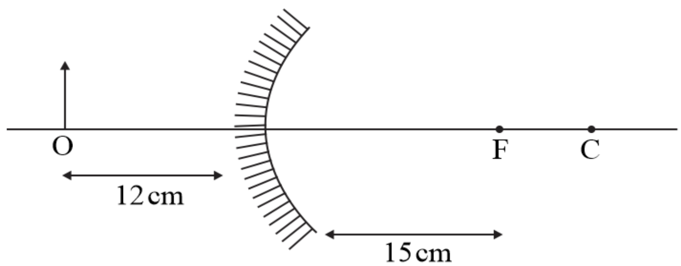
$\therefore$ image height $= m \times$ height of the oject $= -2 \times 2.5 = -5$ cm

The screen must be placed at a distance of 54 cm from the mirror on the same side as the object. The image formed is inverted and twice the size of the object.

As the candle is moved close to the mirror, object approaches the focal point. Hence image will approach infinity. Therefore, the screen has to move away from the mirror to get the image on it.

9.2. A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.

Solution:



Given,

The Object distance $u = -12$ cm

Focal length $f = +15$ cm

From the mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow v = \frac{uf}{u - f} = \frac{(-12) \times 15}{(-12) - 15} = 6.7 \text{ cm}$$

Magnification $m = -\frac{v}{u} = -\frac{6.7}{-12} = 0.56$

The image will be formed at a distance of 6.7 cm right to the mirror. Magnification produced is 0.56.

As the needle moved away from the object image approaches the focal point, or image moves away from the mirror. Size of the image decreases continuously as the object moves away from the mirror.

9.3. A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again?

Solution:

Given,

the apparent depth of the needle $d_a = 9.4$ cm

Height of the water or real depth of the object $d_r = 12.5$ cm

From the relation $\frac{d_a}{d_r} = \frac{1}{{}_1\mu_2}$

$$\Rightarrow \frac{9.4}{12.5} = \frac{1}{\mu_w}$$

$$\Rightarrow \mu_w = 1.33$$

∴ the refractive index of the water is 1.33

If water is replaced by a liquid of refractive index 1.63,

$$\frac{d_a}{d_r} = \frac{1}{{}_1\mu_2} \Rightarrow d_a = \frac{12.5}{1.63} = 7.7 \text{ cm}$$

The microscope must be moved by a distance of 9.4 cm − 7.7 cm = 1.7 cm to focus the object again.

9.4. Figures 9.34(a) and (b) show refraction of a ray in air incident at 60° with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is 45° with the normal to a water-glass interface [Fig. 9.34(c)].

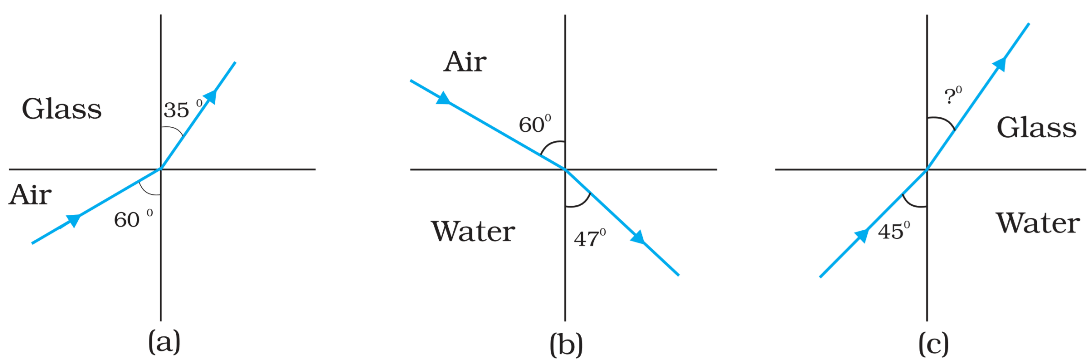


FIGURE 9.34

Solution:

From the figure 9.34(a),

$$\mu_a \sin 60° = \mu_g \sin 35°$$

$$\Rightarrow \mu_{ga} = \frac{\sin 60°}{\sin 35°}$$

$$\Rightarrow \mu_{ga} = 1.51$$

From figure 9.34(b),

$$\mu_a \sin 60° = \mu_w \sin 47°$$

$\Rightarrow \mu_{wa} = \dfrac{\sin 60°}{\sin 47°}$

$\Rightarrow \mu_{wa} = 1.18$

$\Rightarrow \mu_{gw} = \dfrac{1.51}{1.18} = 1.27$

From figure 9.34(c),

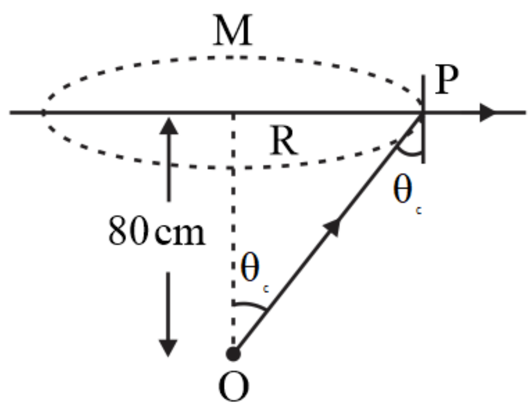
$\mu_w \sin 45° = \mu_g \sin\theta$

$\sin 45° = \mu_{gw} \sin\theta$

$\Rightarrow \theta \simeq 34°$

9.5. A small bulb is placed at the bottom of a tank containing water to a depth of 80 cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source.)

Solution:



Light comes out from a circle on the surface of the water, known as the circle of illuminance

The light ray from the source incident with a critical angle on the border of this circle.

From Snell's law,

$\mu_w \sin\theta_c = \mu_a \sin 90°$

$\Rightarrow 1.33 \times \dfrac{R}{\sqrt{R^2 + 80^2}} = 1$

$\Rightarrow R = 91 \text{ cm} = 91 \text{ m}$

Area$= 2.6 \text{ m}^2$

9.6. A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be 40°. What is the refractive index of the material of the prism? The refracting angle of the prism is 60°. If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.

Solution:

The angle of minimum deviation is related to the relative refractive index of the prism as

$$\mu_{g,medium} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

If the prism is placed in the air

$$\mu_{g,a} = \frac{\sin\left(\frac{60^\circ+40^\circ}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)} \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

$$\mu_{g,a} \simeq 1.53$$

If the prism is placed in water

$$\Rightarrow \mu_{g,w} = \frac{\mu_{g,a}}{\mu_{w,a}} = \frac{\sin\left(\frac{60^\circ+\delta_m}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)}$$

$$\Rightarrow \frac{\mu_{g,a}}{1.33} = \frac{\sin\left(\frac{60^\circ+\delta_m}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

From (i) and (ii)

$$1.33 = \frac{\sin\left(\frac{60^\circ+40^\circ}{2}\right)}{\sin\left(\frac{60^\circ+\delta_m}{2}\right)}$$

$$\sin\left(\frac{60^\circ+\delta_m}{2}\right) = \frac{\sin 50^\circ}{1.33} = 0.56$$

$$\Rightarrow \delta_m \simeq 10^\circ$$

∴ Refractive index of the prism ≃ 1.53 and minimum deviation when placed in water is 10°

9.7. Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20 cm?

Solution:

From the lens makers formula

$$\frac{1}{f} = (\mu_{g,medium} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For convex lens,

$R_1 = R$ and $R_2 = -R$

$$\Rightarrow \frac{1}{20} = (1.55 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

$\Rightarrow R = 22$ cm

∴ The radius of curvature required is 22 cm.

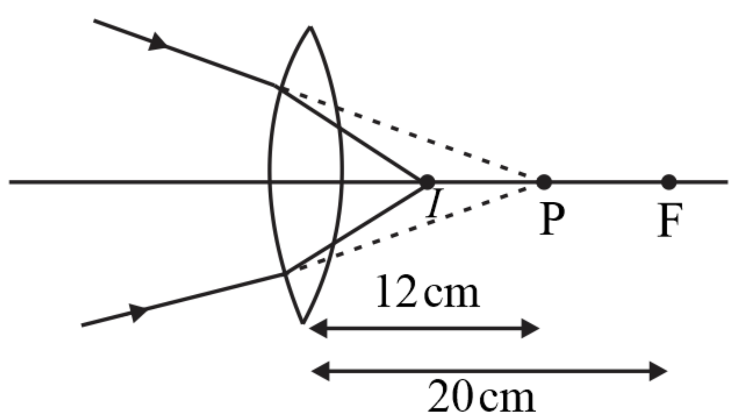
9.8. A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is

(a) a convex lens of focal length 20 cm, and

(b) a concave lens of focal length 16 cm?

Solution:

(a)



Here the object is virtual

From the diagram, Object distance $u = 12$ cm

Focal length $f = 20$ cm

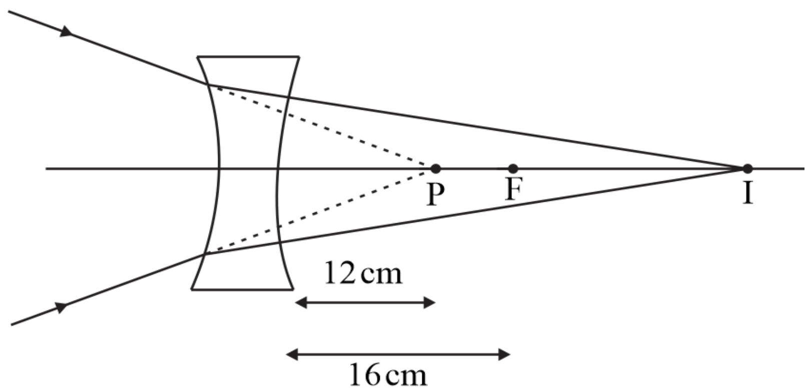
From the thin lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow v = \frac{uf}{u+f} = \frac{12 \times 20}{12 + 20} = 7.5 \text{ cm}$$

The image formed is real and located at a distance of 7.5 cm right side from the lens

(b)



Here the object is virtual

From the diagram, Object distance $u = 12$ cm

Focal length $f = -16$ cm

From the thin lens formula,

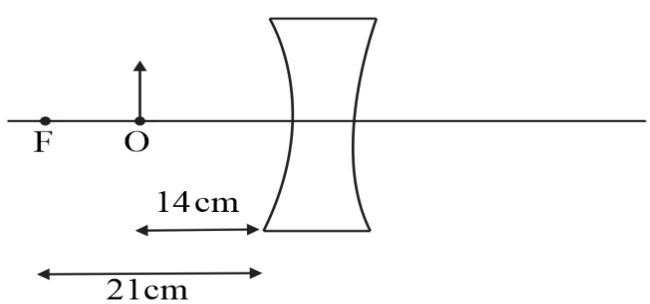
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow v = \frac{uf}{u+f} = \frac{12 \times (-16)}{12 - 16} = 48 \text{ cm}$$

The image formed is real and located at a distance of 48 cm right side from the lens

9.9. An object of size 3.0 cm is placed 14 cm in front of a concave lens of focal length 21 cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?

Solution:



Given,

Object distance $u = -14$ cm

Focal length $f = -21$ cm

From the thin lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow v = \frac{uf}{u+f} = \frac{(-14)\times(-21)}{-14-21} = -8.4 \text{ cm}$$

Magnification $m = \frac{v}{u} = \frac{-8.4}{-14} = 0.6$

$\Rightarrow$ size of the image $= 0.6 \times 3.0 = 1.8$ cm

Image is formed at a distance 8.4 cm left to the lens. The image formed is virtual, erect, diminished to the size 1.8 cm.

As the object moves away from the lens, the image moves towards the focal point $(u \to \infty \Rightarrow v \to f)$ and the magnification approaches zero $(m \to 0)$

9.10. What is the focal length of a convex lens of focal length 30 cm in contact with a concave lens of focal length 20 cm? Is the system a converging or a diverging lens? Ignore thickness of the lenses.

Solution:

Given

The focal length of the convex lens $f_1 = 30$ cm

The focal length of the concave lens $f_2 = -20$ cm

Combined lens focal length f_{eq} is related as

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\Rightarrow \frac{1}{f_{eq}} = \frac{1}{30} + \frac{1}{-20} = -\frac{1}{60}$$

$\Rightarrow f_{eq} = -60$ cm

$\therefore$ The combined lens acts as a diverging lens with a focal length of 60 cm.

9.11. A compound microscope consists of an objective lens of focal length 2.0 cm and an eyepiece of focal length 6.25 cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at

(a) the least distance of distinct vision (25cm), and

(b) at infinity? What is the magnifying power of the microscope in each case?

Solution:

Given,

The focal length of the objective lens $f_o = 2$ cm

The focal length of the eyepiece $f_e = 6.25$ cm

(a) To form the final image at 25 cm from the eyepiece, $v_e = -25$ cm

From thin lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{-25} - \frac{1}{u_e} = \frac{1}{6.25}$$

$\Rightarrow u_e = -5$ cm

Image distance to the objective $v_o = 15 - 5 = 10$ cm

From thin lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{10} - \frac{1}{u_o} = \frac{1}{2}$$

$u_o = -2.5$ cm

Hence object must be placed at a distance of 2.5 cm from the objective

The magnifying power of the compound microscope

$$m = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right) = \frac{10}{2.5}\left(1 + \frac{25}{6.25}\right) = 20$$

(b) To form the final image at ∞, $v_e = \infty$

From thin lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{\infty} - \frac{1}{u_e} = \frac{1}{6.25}$$

$\Rightarrow u_e = -6.25$ cm

Image distance to the objective $v_o = 15 - 6.25 = 8.75$ cm

From thin lens formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{8.75}-\frac{1}{u_o}=\frac{1}{2}$$

$u_o = -2.59$ cm

Hence object must be placed at a distance of 2.59 cm from the objective

The magnifying power of the compound microscope $m=\frac{v_o}{u_o}\left(\frac{D}{f_e}\right)=\frac{8.75}{2.59}\left(\frac{25}{6.25}\right)=13.5$

9.12. A person with a normal near point (25 cm) using a compound microscope with objective of focal length 8.0 mm and an eyepiece of focal length 2.5 cm can bring an object placed at 9.0 mm from the objective in sharp focus. What is the separation between the two lenses? Calculate the magnifying power of the microscope.

Solution:

Given,

The focal length of the objective lens $f_o = 8$ mm $= 0.8$ cm

The focal length of the eyepiece $f_e = 2.5$ cm

Object distance from the objective $u_o = -9$ mm $= -0.9$ cm

Let the image formed by the objective is at a distance v_o

From thin lens formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v_o}-\frac{1}{-0.9}=\frac{1}{0.8}$$

$\Rightarrow v_o = 7.2$ cm

Image distance for the eyepiece $v_e = -25$ cm

From thin lens formula

$$\frac{1}{-25}-\frac{1}{u_e}=\frac{1}{2.5}$$

$\Rightarrow u_e = -2.27$ cm

$\therefore$ the separation between the two lenses $= v_o + |u_e| = 7.2 + 2.27 = 9.47$ cm

Magnifying power $m = \frac{v_o}{|u_o|}\left(1 + \frac{D}{f_e}\right) = \frac{7.2}{0.9}\left(1 + \frac{25}{2.5}\right) = 88$

9.13. A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6.0 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece?

Solution:

Given,

The focal length of the objective $f_o = 144$ cm

The focal length of the eyepiece $f_e = 6.0$ cm

The magnifying power of the telescope $m = \frac{f_o}{f_e} = \frac{144}{6.0} = 24$

Length of the telescope $L = f_o + f_e = 144 + 6.0 = 150$ cm

9.14. (a) A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece of focal length 1.0 cm is used, what is the angular magnification of the telescope?

(b) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is 3.48×10^6 m, and the radius of lunar orbit is 3.8×10^8 m.

Solution:

(a) Given,

The focal length of the objective $f_o = 15$ m $= 1500$ cm

The focal length of the eyepiece $f_e = 1.0$ cm

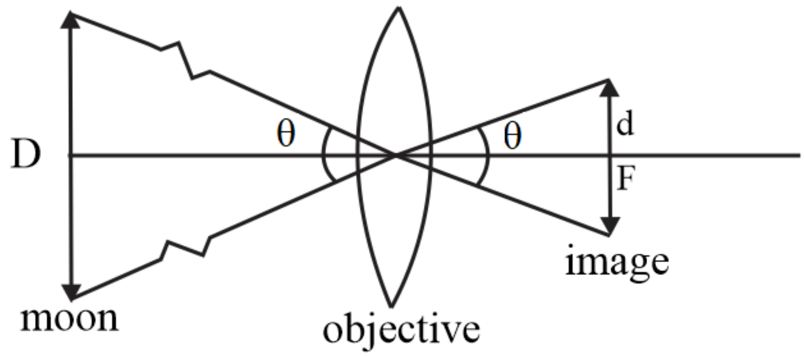
Angular magnification $m = \frac{f_o}{f_e} = \frac{1500}{1} = 1500$

(b) Given,

The diameter of the moon $D = 3.48 \times 10^6$ m

The radius of the lunar orbit $R = 3.8 \times 10^8$ m

The angle subtended by the moon's diameter is equal to the angle subtended by its image diameter at the objective lens



$$\frac{D}{R} = \frac{d}{f_o}$$

$$\Rightarrow d = \frac{3.48 \times 10^6}{3.8 \times 10^8} \times 15 \text{ m} = 13.7 \text{ cm}$$

The diameter of the image formed $d = 13.7$ cm

9.15. Use the mirror equation to deduce that:

(a) an object placed between f and $2f$ of a concave mirror produces a real image beyond $2f$.

(b) a convex mirror always produces a virtual image independent of the location of the object.

(c) the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.

(d) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image. [Note: This exercise helps you deduce algebraically properties of images that one obtains from explicit ray diagrams.]

Solution:

(a) Given that the object is placed between f and $2f$ $\Rightarrow 2f < u < f$ and $u < 0, f < 0$

From the mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow v = \frac{uf}{u - f}$$

If $u = -f$

$$v = \frac{(-f)(-f)}{-f-(-f)} = \infty$$

If $u = -2f$

$$v = \frac{(-2f)(-f)}{-2f-(-f)} = -2f$$

Hence the image will be formed beyond $2f$. The image formed is real and inverted.

(b) From the mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow v = \frac{uf}{u-f}$$

For convex mirror $f > 0$ and $u < 0$

$\Rightarrow v > 0$

$\therefore$ The image always formed backside of the mirror or image formed is virtual.

(c) Magnification $m = -\frac{v}{u} = \frac{f}{f-u}$

For convex mirror $f > 0$ and $u < 0$ $\Rightarrow 0 < \frac{f}{f-u} < 1$ or $0 < m < 1$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

As $u < 0$

$$\frac{1}{v} > \frac{1}{f}$$

Or $v < f$

Thus, the image produced is diminished in size and located between pole and focus

(d) In this case,

$u < 0$, $f < 0$ and $u < f$

From mirror formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{v}=\frac{1}{f}-\frac{1}{u}>0$$

Or $v>0$

Magnification $m=-\frac{v}{u}=\frac{f}{f-u}>1$

Hence, an object placed between the focus and pole of a concave mirror produces a virtual and enlarged image.

9.16. A small pin fixed on a table top is viewed from above from a distance of 50 cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?

Solution:

Given,

The thickness of the glass slab $t=15$ cm

Refractive index of the glass $\mu=1.5$

Shift in the object position due to the slab $s=t\left(1-\frac{1}{\mu}\right)$

$$\Rightarrow s=15\left(1-\frac{1}{1.5}\right)=5 \text{ cm}$$

The shift is independent of the location of the slab for a small angle of incidence.

9.17. (a) Figure 9.35 shows a cross-section of a 'light pipe' made of a glass fibre of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflections inside the pipe take place, as shown in the figure.

(b) What is the answer if there is no outer covering of the pipe?

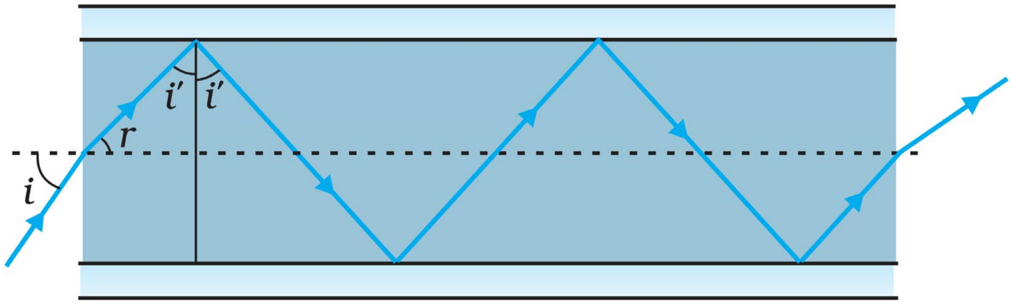


FIGURE 9.35

Solution:

(a) For total internal reflection, near the wall $i' > \theta_c$

$$\sin\theta_c = \frac{1}{\mu_{21}} = \frac{1.44}{1.68} = 0.86$$

$\theta_c = 59°$

$\Rightarrow i' > 59°$

$\Rightarrow r < 31°$

From Snell's law

$1 \times \sin i = 1.68 \times \sin r$

As $r < 31° \;\Rightarrow \sin i < 1.68 \times \sin 31°$

$i < 60°$

Therefore, the range of angle of incidence with the axis is $0 < i < 60°$

(b) If there is no outer covering,

For total internal reflection, near the wall $i' > \theta_c$

$$\sin\theta_c = \frac{1}{\mu_{21}} = \frac{1}{1.68} = 0.60$$

$\theta_c = 36.5°$

$\Rightarrow i' > 36.5°$

$\Rightarrow r < 53.5°$

From Snell's law, for the angle of incidence of 90°

$1 \times \sin 90° = 1.68 \times \sin r$

$\Rightarrow r = 36.5°$ and$i' = 53.5°$ Which is greater than θ_c.

This gives the upper limit as 90° and all the incident rays suffer total internal reflection

9.18. Answer the following questions:

(a) You have learnt that plane and convex mirrors produce virtual images of objects. Can they produce real images under some circumstances? Explain.

(b) A virtual image, we always say, cannot be caught on a screen. Yet when we 'see' a virtual image, we are obviously bringing it on to the 'screen' (i.e., the retina) of our eye. Is there a contradiction?

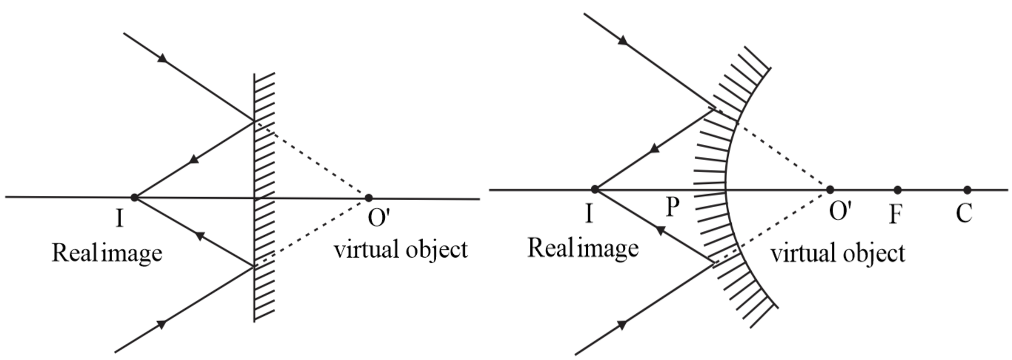
(c) A diver under water, looks obliquely at a fisherman standing on the bank of a lake. Would the fisherman look taller or shorter to the diver than what he actually is?

(d) Does the apparent depth of a tank of water change if viewed obliquely? If so, does the apparent depth increase or decrease?

(e) The refractive index of diamond is much greater than that of ordinary glass. Is this fact of some use to a diamond cutter?

Solution:

(a) The plane mirror always forms a real image for a virtual object, irrespective of object position. The convex mirror forms a real image of a virtual object if it is between pole and focus.



(b) When we see the virtual image, it acts as an object for our eye lens. Eye lens forms a real image of it on the retina. Hence there is no contradiction.

(c) For the diver the fisherman appears taller than he actually is.

(d) Apparent depth decreases if viewed obliquely. For the near normal view apparent depth is maximum.

(e) The diamond refractive index is about 2.42, much bigger than the regular glass index (about 1.5). Diamond's critical angle is about 24°, much lower than glass. A skilled diamond cutter takes advantage of the wider range of incidence angles (in the diamond), 24° to 90°, to guarantee that light entering the diamond is fully reflected from many faces before getting out to produce a sparkling effect.

9.19. The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 3 m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose?

Solution:

Give that, the separation between object and the image is 3 m

Let the object distance from the lens be x, this implies that the image distance is $3 - x$. Let the required focal length be f.

$u = -x$

$v = 3 - x$

$f = f$

From the thin lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{3 - x} - \frac{1}{-x} = \frac{1}{f}$$

$$\Rightarrow f = x - \frac{x^2}{3}$$

To find the maximum of f, $\frac{df}{dx} = 0$

$$\Rightarrow \frac{df}{dx} = 1 - \frac{2x}{3} = 0$$

$$\Rightarrow x = \frac{3}{2} \text{ m}$$

And $f = \frac{3}{2} - \frac{\left(\frac{3}{2}\right)^2}{3} = 0.75$ m

9.20. A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. Determine the focal length of the lens.

Solution:

The separation between the object and the image $D = 90$ cm

The separation between two locations of the object $d = 20$ cm

Let the object distance from the lens be x, this implies that the Image distance is $D - x$

$u = -x$

$v = D - x$

$f = f$

From the thin lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{D - x} - \frac{1}{-x} = \frac{1}{f}$$

$$\Rightarrow x^2 - Dx + Df = 0$$

$$\Rightarrow x = \frac{D \pm \sqrt{D^2 - 4Df}}{2}$$

⇒The two positions of the object are, $\frac{D+\sqrt{D^2-4Df}}{2}$ and $\frac{D-\sqrt{D^2-4Df}}{2}$

The separation between them $= \sqrt{D^2 - 4Df} = d$

$$\Rightarrow f = \frac{D^2 - d^2}{4D} = \frac{90^2 - 20^2}{4 \times 90} = 21.4 \text{ cm}$$

The focal length of the lens $f = 21.4$ cm

9.21. (a) Determine the 'effective focal length' of the combination of the two lenses in Exercise 9.10, if they are placed 8.0 cm apart with their principal axes coincident. Does the answer depend on which side of the combination a beam of parallel light is incident? Is the notion of effective focal length of this system useful at all?

(b) An object 1.5 cm in size is placed on the side of the convex lens in the arrangement (a) above. The distance between the object and the convex lens is 40 cm. Determine the magnification produced by the two-lens system, and the size of the image.

Solution:

(a) The focal length of the convex lens = 30 cm

The focal length of the concave lens = −20 cm

The separation between the lenses $d = 8.0$ cm

Consider a parallel beam of light incident on the first lens

$u_1 = \infty$

$f_1 = f_1$

$v_1 = v_1$

From the thin lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow v_1 = f_1$$

This image acts as an object for the second lens.

$u_2 = -(d - v_1) = -(d - f_1)$

$f_2 = f_2$

$v_2 = v_2$

From the thin lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v_2} + \frac{1}{d - f_1} = \frac{1}{f_2}$$

$$\Rightarrow v_2 = \frac{f_2(d - f_1)}{d - f_1 - f_2}$$

If the parallel beam of light first incident on the convex lens,

$f_1 = 30$ cm

$f_2 = -20$ cm

$$\Rightarrow v_2 = \frac{f_2(d - f_1)}{d - f_1 - f_2} = \frac{-20(8 - 30)}{8 - 30 + 20} = -220 \text{ cm}$$

The parallel incident beam appears to diverge from a point $(220 - 4) = 216$ cm from the centre of the two-lens system.

If the parallel beam of light first incident on the concave lens,

$f_1 = -20$ cm

$f_2 = 30$ cm

$$\Rightarrow v_2 = \frac{f_2(d - f_1)}{d - f_1 - f_2} = \frac{30(8 + 20)}{8 + 20 - 30} = -420 \text{ cm}$$

The parallel incident beam appears to diverge from a point $(420 - 4) = 416$ cm from the centre of the two-lens system.

Hence the answer depends on the side of the combination on which a beam of parallel light is incident. Therefore, the notion of the effective focal length of this system not useful.

(b) Size of the object $h_o = 1.5$ cm

Distance between the object and convex lens is 40 cm

For the convex lens

$u = -40$ cm

$f = 30$ cm

From the thin lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$v = \frac{f \times u}{f + u} = \frac{30 \times (-40)}{30 - 40} = 120 \text{ cm}$$

Magnification produced by convex lens $m_1 = \frac{v}{u} = \frac{f}{f+u} = \frac{30}{30-40} = -3$

For the concave lens

$u = 120 - 8 = 112$ cm

$f = -20$ cm

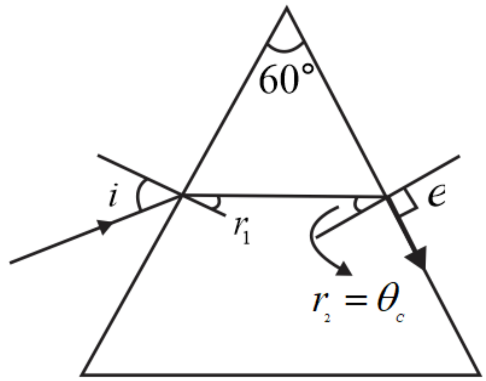
Magnification produced by convex lens $m_1 = \frac{v}{u} = \frac{f}{f+u} = \frac{-20}{-20+112} = -\frac{20}{92}$

Net magnification produced $m = 0.652$

Size of the image $h_i = 0.652 \times 1.5 = 0.98$ cm

9.22. At what angle should a ray of light be incident on the face of a prism of refracting angle 60° so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524.

Solution:



Given,

The refracting angle of prism $A = 60$

Refractive index of the prism $\mu = 1.524$

At the second surface, angle of incidence r_2 must be greater than the critical angle θ_c

$$\theta_c = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{1}{1.524}\right) = 41°$$

$\therefore r_2 > 41° \Rightarrow r_1 < 19°$ ($\because r_1 + r_2 = A = 60°$)

From Snell's law,

$\sin i = \mu \sin r_1 \Rightarrow \sin i < 1.524 \sin 19°$

$\Rightarrow i < 30°$ (approximately)

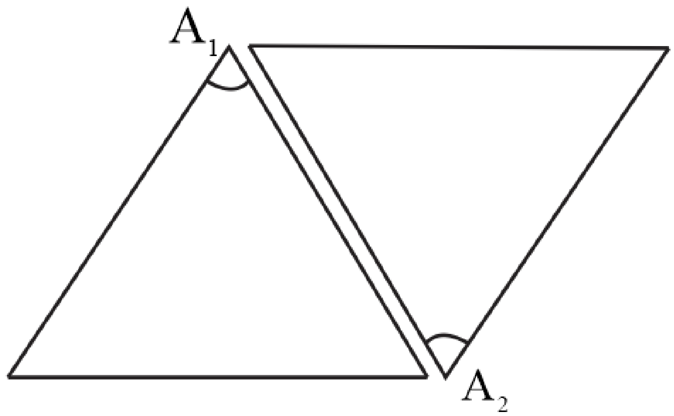
Therefore, the beam will be total internal reflected at the second surface if the angle of incidence at the first surface is lesser than 30°

9.23. You are given prisms made of crown glass and flint glass with a wide variety of angles. Suggest a combination of prisms which will

(a) deviate a pencil of white light without much dispersion.

(b) disperse (and displace) a pencil of white light without much deviation.

Solution:



(a) Combine two prisms inverted as shown in figure without a gap between them. Pencil of white light falls on the left side of the first prism which gets dispersed and deviated. Dispersed light falls on the left side of the second prism in which, it will recombine and pencil of white light comes out of the second surface.

(b) Combine two prisms inverted as shown in figure without a gap between them. The angle of flint glass prism should increased so that the deviation produced by both prisms is equal. The combination disperse the pencil of white light without much deviation.

9.24. For a normal eye, the far point is at infinity and the near point of distinct vision is about 25 cm in front of the eye. The cornea of the eye provides a converging power of about 40 dioptres, and the least converging power of the eye-lens behind the cornea is about 20 dioptres. From this rough data estimate the range of accommodation (i.e., the range of converging power of the eye-lens) of a normal eye.

Solution:

Given,

The near point of distinct vision $d = 25$ cm

Converging power of the cornea $P_1 = 40$ diopters

Least converging power of the eye-lens $P_{2,min} = 20$ diopters

Converging power of the eye-lens is least when the object is at infinity

Total power $P_{min} = 40 + 20 = 60$ diopters

Focal length $f = \frac{1}{P_{min}} \times 100 \text{ cm} = \frac{1}{60} \times 100 \text{ cm} = \frac{5}{3}$ cm

Object distance $u = \infty$

From the thin lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow v=f=\frac{5}{3}\text{ cm}$$

As the image is always formed on the retina, the distance between the lens and the retina $=\frac{5}{3}$ cm

When the object is at the near point

$u=-25$ cm

$v=\frac{5}{3}$ cm

Let the effective focal length be f

From the thin lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow\frac{1}{f}=\frac{3}{5}+\frac{1}{25}=\frac{16}{25}$$

Maximum converging power $P_{max}=\frac{1}{f(in\text{ cm})}\times 100$ diopters $=64$ diopters

Hence converging power of the eye lens $=64-40=24$ diopters

∴ Range of accommodation is 20 diopters to 24 diopters.

9.25. Does short-sightedness (myopia) or long-sightedness (hypermetropia) imply necessarily that the eye has partially lost its ability of accommodation? If not, what might cause these defects of vision?

Solution:

It is not necessary that the eye with short-sightedness or long-sightedness imply a partial loss in the ability of accommodation. Short-sightedness or myopia may occur when the eye-balls get elongated from front to back and Hypermetropia may occur when eye-ball get shortened. The defect of vision in which the eye has partially lost its ability of accommodation is termed as presbyopia.

9.26. A myopic person has been using spectacles of power -1.0 dioptre for distant vision. During old age he also needs to use separate reading glass of power $+2.0$ dioptres. Explain what may have happened.

Solution:

Given

The power of spectacles of the person $P_1 = -1.0$ diopters

The focal length of the lens used $f_1 = \frac{1}{P_1} \times 100 = -100$ cm

This implies that, he has a far point of 100 cm and normal near point of 25 cm

In his old age he needed separate reading glasses of power $+2.0$ diopters

⇒The focal length of the lens $f_2 = \frac{1}{2} \times 100 = 50$ cm

This implies that in his old age his near-point increased from 25 cm

$u = -25$ cm

$v = v$

$f = 50$ cm

From the thin lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-25} = \frac{1}{50}$$

$u = -50$ cm

Therefore, his near-point has changed from 25 cm to 50 cm. In the old age he partially lost the ability of accommodation. This defect is known as presbyopia. Due to this he cannot see the objects placed in front of him clearly up to the distance 50 cm.

9.27. A person looking at a person wearing a shirt with a pattern comprising vertical and horizontal lines is able to see the vertical lines more distinctly than the horizontal ones. What is this defect due to? How is such a defect of vision corrected?

Solution:

In the given situation the person can see the vertical lines more distinctly than the horizontal ones. The type of defect is called astigmatism. This occurs when the cornea is not spherical in shape. In the given situation he is not able to focus the horizontal lines, hence the curvature of the cornea in the horizontal plane is not sufficient compared to the curvature in the vertical plane. Astigmatism can be corrected by using a cylindrical lens of the desired radius of curvature with an appropriately directed axis

9.28. A man with normal near point (25 cm) reads a book with small print using a magnifying glass: a thin convex lens of focal length 5 cm.

(a) What is the closest and the farthest distance at which he should keep the lens from the page so that he can read the book when viewing through the magnifying glass?

(b) What is the maximum and the minimum angular magnification (magnifying power) possible using the above simple microscope?

Solution:

(a) The focal length of the lens $f = 5$ cm

The image can be formed between 25 cm and infinity

From the thin lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

For $v = -25$ cm

$$\Rightarrow u = \frac{fv}{f - v} = \frac{5(-25)}{5 + 25} = -4.16 \text{ cm}$$

For $v = \infty$, $u = -5$ cm

Hence the closest distance he should keep the lens from the page is 4.167 cm, and the farthest distance is 5 cm

(b) Minimum magnifying power is when the image formed at infinity.

$$m_{min} = \frac{D}{f} = \frac{25 \text{ cm}}{5 \text{ cm}} = 5$$

Maximum magnifying power is when the image is formed at the near-point

$$m_{max} = 1 + \frac{D}{f} = 6$$

9.29. A card sheet divided into squares each of size 1 mm^2 is being viewed at a distance of 9 cm through a magnifying glass (a converging lens of focal length 10 cm) held close to the eye.

(a) What is the magnification produced by the lens? How much is the area of each square in the virtual image?

(b) What is the angular magnification (magnifying power) of the lens?

(c) Is the magnification in (a) equal to the magnifying power in (b)? Explain.

(Note: Data in the question is modified to get the proper answers)

Solution:

(a) Given,

The area of each square $A = 1 \text{ mm}^2$

The distance between card sheet and the lens $u = -9$ cm

The focal length of the lens $f = 10$ cm

From thin lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$v = \frac{uf}{u + f} = \frac{-9 \times 10}{-9 + 10} = -90 \text{ cm}$$

Magnification produced $m = \frac{v}{u} = \frac{-90}{-9} = 10$

⇒ length and breadth of the square increases 10 times

Area of each square box $= 10 \text{ mm} \times 10 \text{ mm} = 100 \text{ mm}^2$

(b) Angular magnification $m' = \frac{D}{|u|} = \frac{25}{9} = 2.8$

(c) No. Magnification and angular magnification of an optical instrument are two separate things. Angular magnification is defined as the ratio of the angular size of the object to the angular size of the object when placed at near point (25 cm).

Magnification $m = \left|\frac{v}{u}\right|$ and angular magnification $m' = \frac{25}{|u|}$

Their magnitudes are equal only when the image of an object is formed at near point (25 cm)

9.30. (a) At what distance should the lens be held from the figure in Exercise 9.29 in order to view the squares distinctly with the maximum possible magnifying power?

(b) What is the magnification in this case?

(c) Is the magnification equal to the magnifying power in this case? Explain.

Solution:

(a) Given,

The area of each square $A = 1 \text{ mm}^2$

The focal length of the lens $f = 10$ cm

Magnifying power is maximum when the image of the given object is formed at the near point (25 cm)

⇒ $v = -25$ cm

From thin lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$u = \frac{vf}{f - v} = \frac{-25 \times 10}{10 + 25} = -7.14 \text{ cm}$$

the magnifying power of the lens is maximum when the sheet is placed at 7.14 cm in front of the lens

(b) Magnification $m = \frac{v}{u} = \frac{-25}{-7.14} = 3.5$

(c) Magnifying power $m' = \frac{25}{|u|} = \frac{25}{7.14} = 3.5$

Magnitudes of magnifying power and magnification are equal in this case. If the image of an object is formed at the near point (25 cm), magnitudes of magnifying power and magnification are equal.

9.31. What should be the distance between the object in Exercise 9.30 and the magnifying glass if the virtual image of each square in the figure is to have an area of 6.25 mm^2. Would you be able to see the squares distinctly with your eyes very close to the magnifier? [Note: Exercises 9.29 to 9.31 will help you clearly understand the difference between magnification in absolute size and the angular magnification (or magnifying power) of an instrument.]

Solution:

Given,

The area of each square $= 6.25 \text{ mm}^2$

$\Rightarrow$ The side length of the square $= 2.5$ mm

Magnification required $m = \frac{v}{u} = \frac{2.5 \text{ mm}}{1 \text{ mm}} = 2.5$

$\Rightarrow v = 2.5u$

From the thin lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{2.5u} - \frac{1}{u} = \frac{1}{10}$$

$\Rightarrow u = -6$ cm

And $v = -15$ cm, which is less than near-point (25 cm)

Hence, we cannot see the square distinctly with our eyes very close to the magnifier

9.32. Answer the following questions:

(a) The angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image produced by a magnifying glass. In what sense then does a magnifying glass provide angular magnification?

(b) In viewing through a magnifying glass, one usually positions one's eyes very close to the lens. Does angular magnification change if the eye is moved back?

(c) Magnifying power of a simple microscope is inversely proportional to the focal length of the lens. What then stops us from using a convex lens of smaller and smaller focal length and achieving greater and greater magnifying power?

(d) Why must both the objective and the eyepiece of a compound microscope have short focal lengths?

(e) When viewing through a compound microscope, our eyes should be positioned not on the eyepiece but a short distance away from it for best viewing. Why? How much should be that short distance between the eye and eyepiece?

Solution:

(a) The angular size of the object and the image are the same. The advantage of the magnifying glass is that, the object can be placed closer than near point (25 cm). Hence it gives a larger angular size than the object placed at near point. In this sense a magnifying glass provides angular magnification.

(b) As the angle subtended by the image at the eye is slightly less than the angle subtended at the lens Angular magnification decreases a little. If the image is at a very large distance away this effect is negligible.

(c) The focal length of a convex lens cannot be decreased to a greater amount. If we reduce the focal length, spherical and chromatic aberrations will become more pronounced. So, in practice one cannot get a magnifying power of more than 3 for a simple convex lens. However, using a system of lenses which corrects the aberration one can increase this limit by a factor 10.

(d) The magnifying power of the eyepiece $m_e = \left(1 + \frac{D}{f_e}\right)$

Hence magnifying power of eyepiece increases with smaller f_e

Magnification of the objective can be written as $\frac{v_o}{|u_o|}$

Objects are placed close to the focal point of the objective. This gives $|u_o| \approx f_o$. Hence to increase the magnification, the focal length of the eyepiece is maintained small.

(e) The image of the objective in the eyepiece is called eye-ring. If we place our eye too close to the eyepiece, our eye cannot collect much of the light and also the field of view gets reduced. If we position our eye at the eye ring which is slightly above the eyepiece, the area of the pupil of our eye is greater or equal to the area of the eye ring and our eyes will collect all the light refracted by the objective.

The precise location of the eye-ring naturally depends on the separation between the objective and the eyepiece.

9.33. An angular magnification (magnifying power) of 30X is desired using an objective of focal length 1.25 cm and an eyepiece of focal length 5 cm. How will you set up the compound microscope?

Solution:

Given,

The focal length of the objective $f_o = 1.25$ cm

The focal length of the eyepiece $f_e = 5$ cm

The magnifying power of a compound microscope $m = m_o \times m_e$

Assuming near point adjustment,

$$m = \frac{v_o}{-u_o}\left(1 + \frac{D}{f_e}\right) = 30 \text{ cm}$$

$$\Rightarrow \frac{v_o}{-u_o}\left(1 + \frac{25}{5}\right) = 30$$

$$\Rightarrow v_o = -5u_o$$

From the thin lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v_o} + \frac{5}{v_o} = \frac{1}{1.25}$$

$$\Rightarrow v_o = 7.5 \text{ cm, and } u_o = -1.5 \text{ cm}$$

Similarly, for the eyepiece,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{-25} + \frac{1}{u_e} = \frac{1}{5}$$

$$\Rightarrow u_e = 4.17 \text{ cm}$$

Therefore, the separation between objective and the eyepiece $L = 7.5 + 1.17 = 11.67$ cm

And the object must be placed at 1.5 cm to get the given magnification.

9.34. A small telescope has an objective lens of focal length 140 cm and an eyepiece of focal length 5.0 cm. What is the magnifying power of the telescope for viewing distant objects when

(a) the telescope is in normal adjustment (i.e., when the final image is at infinity)?

(b) the final image is formed at the least distance of distinct vision (25cm)?

Solution:

Given,

The focal length of the objective $f_o = 140$ cm

The focal length of the eyepiece $f_e = 5.0$ cm

(a) For normal adjustment,

Magnifying power $m = \frac{f_o}{f_e} = \frac{140}{5.0} = 28$

(b) For near-point adjustment,

Magnifying power $m = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right) = \frac{140}{5.0}\left(1 + \frac{5.0}{25}\right) = 33.6$

9.35. (a) For the telescope described in Exercise 9.34 (a), what is the separation between the objective lens and the eyepiece?

(b) If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens?

(c) What is the height of the final image of the tower if it is formed at 25 cm?

Solution:

(a) From Exercise 9.34 (a),

The focal length of the objective $f_o = 140$ cm

The focal length of the eyepiece $f_e = 5.0$ cm

For normal adjustment, the separation between the objective and the eye-piece

$L = f_o + f_e$

$L = 140 + 5 = 145$ cm

(b) Given,

The Height of the tower $h_o = 100$ m

The distance of the tower $u_o = 3$ km $= 3000$ m

The angle subtended by the tower at the objective = Angle subtended by the image of the objective

$\Rightarrow$ Height of the image formed by the objective $h_1 = f_o \times \frac{h_o}{u_o} = 140 \times \frac{100}{3000} = 4.7$ cm

(c) For the eyepiece,

$$\frac{1}{u} - \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{u_e} - \frac{1}{-25} = \frac{1}{5}$$

$$\Rightarrow u_e = 6.5 \text{ cm}$$

The angle subtended by the image of the objective at the eyepiece = Angle subtended by the image of the eyepiece

$$\Rightarrow \frac{4.7 \text{ cm}}{6.5 \text{ cm}} = \frac{h_2}{25 \text{ cm}}$$

$$\Rightarrow h_2 = 18.07 \text{ cm}$$

Height of the final image of the tower = 18.07 cm

9.36. A Cassegrain telescope uses two mirrors as shown in Fig. 9.33. Such a telescope is built with the mirrors 20 mm apart. If the radius of curvature of the large mirror is 220 mm and the small mirror is 140 mm, where will the final image of an object at infinity be?

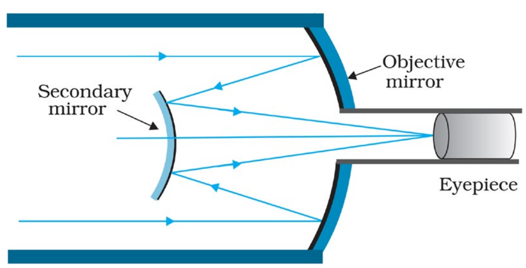


Fig. 9.33

Solution:

Given,

The radius of the curvature of the large mirror $R_1 = 220$ mm

Its focal length = 110 mm = 11 cm

The radius of the curvature of the small mirror $R_1 = -140$ mm

Its focal length = 70 mm = 7 cm

The separation between the mirrors = 20 mm = 2 cm

For the large mirror

$u_1 = \infty$

$f_1 = -11$ cm

From mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v_1} + \frac{1}{\infty} = \frac{1}{-11}$$

$v_1 = -11$ cm

The image formed by the large mirror serves as an object to the small mirror

For the small mirror,

$u_2 = 11 - 2 = 9$ cm

$f_2 = 7$ cm

From mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v_2} + \frac{1}{9} = \frac{1}{7}$$

$v_2 = 31.5$ cm = 315 mm

Hence the final image will be formed at a distance 315 mm left to the small mirror

9.37. Light incident normally on a plane mirror attached to a galvanometer coil retraces backwards as shown in Fig. 9.36. A current in the coil produces a deflection of 3.5° of the mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away?

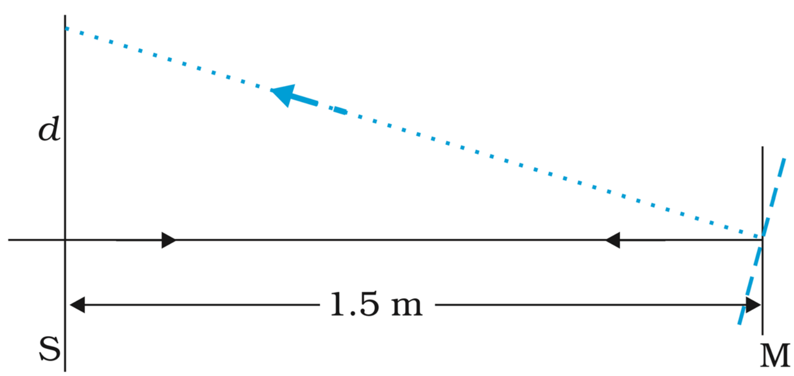


Fig. 9.36

Solution:

Given that the reflection in the mirror $\theta = 3.5° = \left(\frac{\pi}{180} \times 3.5°\right)$ rad

When the mirror is rotated through an angle θ the reflected light beam from it rotates through an angle 2θ

Thus, the angle rotated by the reflected light beam $= 2 \times \left(\frac{\pi}{180} \times 3.5°\right)$ rad

Distance moved by the spot on the screen $d = 1.5 \times 2 \times \left(\frac{\pi}{180} \times 3.5°\right)$ m $=$ 18.3 cm

9.38. Figure 9.37 shows an equiconvex lens (of refractive index 1.50) in contact with a liquid layer on top of a plane mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of the needle from the lens is measured to be 45.0 cm. The liquid is removed and the experiment is repeated. The new distance is measured to be 30.0 cm. What is the refractive index of the liquid?

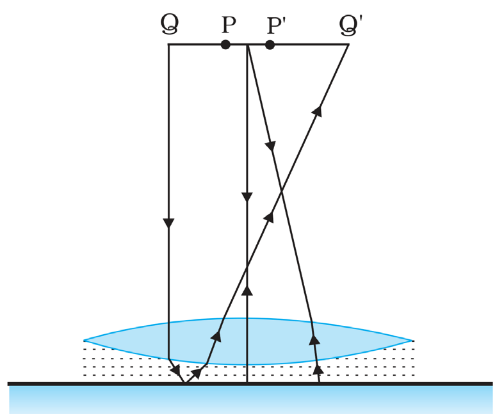


Figure 9.37

Solution:

Given:

The refractive index of the convex lens = 1.50

Let the refractive index of the liquid be μ

In both cases, object distance and image distances are the same. This implies incoming rays are retracing the same path after reflecting from the mirror, or rays from the pin tip going parallel after refracting from the lens and water.

Let the radius of curvature of the equiconvex lens be R

From the lens makers formula $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

The focal length of the convex lens, $\frac{1}{f_1} = (1.50 - 1)\left(\frac{1}{R_1} - \frac{1}{-R}\right) = \frac{1}{R}$

$\Rightarrow f_1 = R$

The focal length of the liquid layer, $\frac{1}{f_2} = (\mu_l - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right) = -\frac{(\mu_l - 1)}{R}$

$$\Rightarrow f_2 = -\frac{(\mu_l - 1)}{R}$$

Case (i): With water

The equivalent focal length of the lens and water layer

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

From lens makers formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{\infty} - \frac{1}{u_1} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{R} - \frac{\mu - 1}{R}$$

$\frac{1}{u_1} = \frac{\mu - 1}{R} - \frac{1}{R}$ …………….. (i)

Case (i): Without water

From lens makers formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{\infty} - \frac{1}{u_2} = \frac{1}{f_1} = \frac{1}{R}$$

$$\frac{1}{u_2} = -\frac{1}{R}$$

$\Rightarrow -u_2 = R = 30$ cm

From equation (i)

$$\frac{1}{-45} = \frac{\mu - 1}{30} - \frac{1}{30}$$

$$\mu = \frac{4}{3} = 1.33$$

Therefore the refractive index of the liquid used is 1.33

♦♦♦