CBSE NCERT Solutions for Class 12 Physics Chapter 11

Back of Chapter Questions

1. Find the

(a) maximum frequency, and

(b) minimum wavelength of X-rays produced by 30 kV electrons.

Solution:

Given,

The potential of the electron is $V = 30\text{kV} = 3 \times 10^4 V$.

The potential energy in electron volts is $E = 3 \times 10^4 \text{eV}$.

(a) Let the maximum frequency of X-rays produced be ν.

The equation for frequency is,

$E = h\nu$

$\Rightarrow \nu = \frac{E}{h}$

Substituting the values,

$$\nu = \frac{1.6 \times 10^{-19} \times 3 \times 10^4}{6.626 \times 10^{-34}} = 7.24 \times 10^{18}\text{Hz}$$

Thus, the maximum frequency of X-rays produced is 7.24×10^{18} Hz.

(b) The equation for minimum wavelength of X-rays produced is

$\lambda = \frac{c}{\nu}$

Substituting the values,

$$\lambda = \frac{3 \times 10^8}{7.24 \times 10^{18}}$$

$$= 4.14 \times 10^{-11} \text{ m}$$

$$= 0.0414 \text{ nm}$$

Thus, the minimum wavelength of X-rays produced is 0.0414 nm.

2. The work function of caesium metal is 2.14 eV. When light of frequency 6×10^{14} Hz is incident on the metal surface, photoemission of electrons occurs. What is the

(a) maximum kinetic energy of the emitted electrons,

(b) Stopping potential, and

(c) maximum speed of the emitted photoelectrons?

Solution:

Given

Here, the work function of caesium metal is 2.14 eV.

The frequency of light is 6×10^{14} Hz.

(a) For the emitted electrons, the maximum kinetic energy is given by the equation,

$$K = h\nu - \phi_0$$

Substituting the values,

$$K = \frac{(6.626 \times 10^{-34})(6 \times 10^{14})}{1.6 \times 10^{-19}} - 2.14$$

$$= 0.345 \text{ eV}$$

Thus, the maximum kinetic energy of the emitted electrons is 0.345 eV.

(b) The stopping potential of the emitted electron is found from the relation,

$$K = eV_0$$

$$\Rightarrow V_0 = \frac{K}{e}$$

Substituting the values,

$$V_0 = \frac{0.345 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-19}} = 0.345 \text{ V}$$

Thus, the stopping potential of the emitted electron is 0.345 V.

(c) The maximum speed of the photoelectrons is found from the relation,

$$v^2 = \frac{2K}{m}$$

$$\Rightarrow v = \sqrt{\frac{2K}{m}}$$

Substituting the values,

$$v = \sqrt{\frac{2 \times 0.345 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}}$$

$$= \sqrt{0.1104 \times 10^{12}}$$

$= 3.323 \times 10^5 \text{ m/s}$

$= 332.3 \text{ km/s}$

Thus, the maximum speed of the emitted photoelectrons is 332.3 km/s.

3. The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?

Solution:

Given

The cutoff voltage is $V_0 = 1.5$ V.

The maximum kinetic energy of the electrons is given by the equation,

$K = eV_0$

Substituting the values,

$K = 1.6 \times 10^{-19} \times 1.5 = 2.4 \times 10^{-19}$ J

Thus, the maximum kinetic energy of the emitted electrons is 2.4×10^{-19} J.

4. Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW.

(a) Find the energy and momentum of each photon in the light beam,

(b) How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section which is less than the target area), and

(c) How fast does a hydrogen atom have to travel to have the same momentum as that of the photon?

Solution:

Given

The wavelength of the monochromatic light is $\lambda = 632.8 \text{ nm} = 632.8 \times 10^{-9}$ m.

The emitted power of the laser is $P = 9.42 \text{ mW} = 9.42 \times 10^{-3}$ W.

The mass of the hydrogen atom is, $m = 1.66 \times 10^{-27}$ kg.

(a) The energy of each photon in the light beam is

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{632.8 \times 10^{-9}}$$

$$= 3.141 \times 10^{-19} \text{ J}$$

The momentum of each photon in the light beam is

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}}{632.8 \times 10^{-9}}$$

$$= 1.047 \times 10^{-27} \text{ kg m/s}$$

Thus, the energy of each photon in the light beam is 3.141×10^{-19} J and the momentum of each photon in the light beam is 1.047×10^{-27} kg m/s.

(b) Let the number of photons arriving per second at the target be n.

The number can be found from the equation for power.

$$P = nE$$

$$\Rightarrow n = \frac{P}{E} = \frac{9.42 \times 10^{-3}}{3.141 \times 10^{-19}} = 3 \times 10^{16} \text{ photons/s}$$

Thus, the number of photons arriving per second at the target is 3×10^{16} photons/s.

(c) Momentum of photon and hydrogen atom is the same.

$$p = 1.047 \times 10^{-27} \text{ kgm/s}$$

The speed of the hydrogen atom is,

$$v = \frac{p}{m} = \frac{1.047 \times 10^{-27}}{1.66 \times 10^{-27}} = 0.621 \text{ m/s}$$

Thus, the speed of the hydrogen atom is 0.621 m/s.

5. The energy flux of sunlight reaching the surface of the earth is 1.388×10^3 W/m^2. How many photons (nearly) per square metre are incident on the Earth per second? Assume that the photons in the sunlight have an average wavelength of 550 nm.

Solution:

Given

The energy flux of sunlight is $\Phi = 1.388 \times 10^3$ W/m^2

Average wavelength of the photon is $\lambda = 550$ nm $= 550 \times 10^{-9}$ m.

The power of sunlight per square metre is $P = 1.388 \times 10^3$ W

Let the number of photons incident per second on earth be n.

$$P = nE$$

$$\Rightarrow n = \frac{P}{E} = \frac{P\lambda}{hc}$$

Substituting the values,

$$n = \frac{(1.388 \times 10^3)(550 \times 10^{-9})}{(6.626 \times 10^{-34})(3 \times 10^8)}$$

$$\Rightarrow n = 3.84 \times 10^{21} \text{ photons/m}^2\text{/s}$$

Thus, the number of photons (nearly) per square metre are incident on the earth per second is 3.84×10^{21} photons.

6. In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be 4.12×10^{-15} V s. Calculate the value of Planck's constant.

Solution:

The slope of cut-off voltage versus frequency of incident light is,

$$\frac{V}{\nu} = 4.12 \times 10^{-15} \text{ V s}$$

The relationship between the cut-off voltage and frequency is given by,

$$h\nu = eV$$

Here,

e is charge of electron

h is the Plank's constant

Therefore, the Plank's constant is,

$$h = e\frac{V}{\nu}$$

$$= 1.6 \times 10^{-19} \text{ C} \times 4.12 \times 10^{-15}$$

$$= 6.59 \times 10^{-34} \text{ J s}$$

Therefore, the value of Plank's constant is 6.59×10^{-34} J s.

7. A 100W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm. (a) What is the energy per photon associated with the sodium light? (b) At what rate are the photons delivered to the sphere?

Solution:

The power of the sodium lamp is, $P = 100$ W

The wavelength of sodium light emitted is,

$$\lambda = 589 \text{ nm} = 589 \times 10^{-9} \text{ m}$$

The Plank's constant, $h = 6.626 \times 10^{-34}$ J s

The speed of light, $c = 3 \times 10^8$ m/s

(a) The energy per photon for the sodium light is,

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34}\text{ Js} \times 3 \times 10^{-8}\text{ m/s}}{589 \times 10^{-9}\text{ m}}$$

$$= 3.37 \times 10^{-19}\text{ J}$$

$$= (3.37 \times 10^{-19}\text{ J}) \times \frac{1\text{ eV}}{1.6 \times 10^{-19}\text{ J}}$$

$$= 2.11\text{ eV}$$

(b) The power is,

$$P = nE$$

Here, n is the number of photons delivered to the sphere.

Therefore, the number of photons delivered to the sphere is,

$$n = \frac{P}{E}$$

$$= \frac{100\text{ W}}{3.37 \times 10^{-19}\text{ J}}$$

$$= 2.96 \times 10^{20}\text{ photons/s}$$

Thus, 2.96×10^{20} photons are delivered to the sphere every second.

8. The threshold frequency for a certain metal is $3.3 \times 1014\ Hz$. If light of frequency 8.2×10^{14} Hz is incident on the metal, predict the cutoff voltage for the photoelectric emission.

Solution:

The threshold frequency of the metal is, $\nu_0 = 3.3 \times 10^{14}$ Hz

The frequency of incident light on metal is, $\nu = 8.2 \times 10^{14}$ Hz

The charge of electron is, $e = 1.6 \times 10^{-19}$ C

The plank's constant is, $h = 6.626 \times 10^{-34}$ Js

The cut off energy is,

$$V_0 = \frac{h(\nu - \nu_0)}{e}$$

$$= \frac{6.626 \times 10^{-34} \text{ Js} \times (8.2 \times 10^{14} \text{ Hz} - 3.3 \times 10^{14} \text{ Hz})}{1.6 \times 10^{-19} \text{ C}}$$

$$= 2.03 \text{ V}$$

Thus, the cut-off voltage for the photoelectric emission is 2.03 V.

9. The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?

Solution:

Given

The work function is 4.2 eV.

The incident wavelength is $\lambda = 330 \text{ nm} = 330 \times 10^{9} \text{ m}$

The energy of the incident photon is

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{330 \times 10^{9} \text{ m}}$$

$$= 6.0 \times 10^{-19} \text{ J}$$

$$= \frac{6.0 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.76 \text{ eV}$$

Here, as the incident radiation has an energy less than the work function of the metal. Thus, no photoelectric emission will take place.

10. Light of frequency 7.21×10^{14} Hz is incident on a metal surface. Electrons with a maximum speed of $6.0 \times 10^{5} \text{ m/s}$ are ejected from the surface. What is the threshold frequency for photoemission of electrons?

Solution:

The frequency of the incident light is,

$$v = 488 \text{ nm} = 488 \times 10^{-9} \text{ m}$$

The maximum speed of the electron is, $v = 6.0 \times 10^{5} \text{ m/s}$

The plank’s constant is, 6.626×10^{-34} Js

The mass of the electron is, 9.1×10^{-31} kg

The kinetic energy is,

$$\frac{1}{2}mv^2 = h(\nu - \nu_0)$$

Here, ν_0 is the threshold frequency. Therefore, the threshold frequency is,

$$\nu_0 = \nu - \frac{mv^2}{2h}$$

$$= 7.21 \times 10^{14}\ \text{Hz} - \frac{(9.1 \times 10^{-31}\ \text{kg})(6.0 \times 10^{5}\ \text{m/s})^2}{2(6.626 \times 10^{-34}\ \text{Js})}$$

$$= 7.21 \times 10^{14}\ \text{Hz} - 2.47 \times 10^{14}\ \text{Hz}$$

$$= 4.74 \times 10^{14}\ \text{Hz}$$

Thus, 4.74×10^{14} Hz is the threshold frequency of the photoemission.

11. Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.

Solution:

The wavelength of the light emitted from Argon laser is,

$$\lambda = 488\ \text{nm} = 488 \times 10^{-9}\ \text{m}$$

The stopping potential of the photoelectron is,

$$V_0 = 0.38\ \text{V}$$

$$= \frac{0.38\ \text{V}}{1.6 \times 10^{-19}\ \text{V/eV}}$$

$$= 0.608 \times 10^{19}\ \text{eV}$$

The Plank's constant is, $h = 6.626 \times 10^{-34}$ Js

The charge of the electron is, $e = 1.6 \times 10^{-19}$ C

The speed of the electron is, $c = 3 \times 10^{8}$ m/s

The work function is,

$$\phi = \frac{hc}{\lambda} - eV_0$$

$$\frac{\left(6.626 \times 10^{-34}\ \text{Js} \times 3 \times 10^{8}\ \text{m/s}\right)}{\left(1.6 \times 10^{-19}\ \text{C}\right)\left(488 \times 10^{-9}\ \text{m}\right)} - \left(1.6 \times 10^{-19}\ \text{C}\right)\left(0.608 \times 10^{19}\ \text{eV}\right)$$

$$= 2.16\ \text{eV}$$

Thus, 2.16 eV is the work function required for the material.

12. Calculate the

(a) momentum, and

(b) de Broglie wavelength of the electrons accelerated through a potential difference of 56 V.

Solution:

The potential difference is, $\mathrm{V} = 56\ \mathrm{V}$

The Plank's constant, $h = 6.6 \times 10^{-34}$ Js

The mass of electron is, $m = 9.1 \times 10^{-31}$ kg

The charge of the electron is, $e = 1.6 \times 10^{-19}\ \mathrm{C}$

(a) The kinetic energy of each electron at equilibrium is,

$$\frac{1}{2}mv^2 = eV$$

Here, v is the velocity.

Therefore, the velocity is,

$$v = \sqrt{\frac{2eV}{m}}$$

$$= \sqrt{\frac{2(1.6 \times 10^{-19}\ \mathrm{C})(56\ \mathrm{V})}{(9.1 \times 10^{-31}\ \mathrm{kg})}}$$

$$= 4.44 \times 10^6\ \mathrm{m/s}$$

The momentum of each accelerated electron is,

$$p = mv$$

$$= (9.1 \times 10^{-31}\ \mathrm{kg})(4.44 \times 10^6\ \mathrm{m/s})$$

$$= 4.04 \times 10^{-24}\ \mathrm{kgm/s}$$

Therefore, $4.04 \times 10^{-24}\ \mathrm{kgm/s}$ is the momentum of each electron.

(b) The de Broglie wavelength of the electron accelerated through a potential is,

$$\lambda = \frac{12.27\mathring{\mathrm{A}}}{\sqrt{V}}$$

Here, V is the potential.

Therefore, de Broglie wavelength of electron is,

$$\lambda = \frac{12.27\ \mathring{\mathrm{A}}}{\sqrt{56\ \mathrm{V}}}$$

$$= \frac{\left(12.27\ \mathring{A}\right)\left(\frac{1\ \text{m}}{10^{10}\ \text{m}}\right)}{\sqrt{56\ \text{V}}}$$

$$= \frac{12.27 \times 10^{-10}\ \text{m}}{\sqrt{56\ \text{V}}}$$

$$= 0.16 \times 10^{-9}\ \text{m}$$

$$= (0.16 \times 10^{-9}\ \text{m})\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right)$$

$$= 0.16\ \text{nm}$$

Thus, 0.16 nm is the de Broglie wavelength of electron.

13. What is the

(a) momentum,

(b) speed, and

(c) de Broglie wavelength of an electron with kinetic energy of 120 eV.

Solution:

Given

Kinetic energy of the electron is $KE = 120\ \text{eV}$.

(a) The speed of the electron is,

$$KE = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2(KE)}{m}}$$

$$= \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 120}{9.1 \times 10^{-31}}}$$

$$= 6.496 \times 10^{6}\ \text{m/s}$$

The momentum of the electron is,

$$p = mv$$

$$= (9.1 \times 10^{-31}\ \text{kg})(6.496 \times 10^{6}\ \text{m/s})$$

$$= 5.91 \times 10^{-24}\ \text{kg}\frac{\text{m}}{\text{s}}$$

Thus, the momentum of the electron is $5.91 \times 10^{-24}\ \text{kg m/s}$.

(b) The speed of the electron is 6.496×10^6 m/s.

(c) De Broglie wavelength of the electron is

$$\lambda = \frac{h}{p}$$

$$= \frac{6.626 \times 10^{-34}}{5.91 \times 10^{-24}}$$

$$= 1.116 \times 10^{-10}\ \mathrm{m}$$

$$= 0.112\ \mathrm{nm}$$

Thus, the De Broglie wavelength of the electron is 0.112 nm.

14. The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which

(a) an electron, and

(b) a neutron would have the same de Broglie wavelength.

Solution:

The wavelength of the sodium light is,

$\lambda = 589\ \mathrm{nm} = 589 \times 10^{-9}\ \mathrm{m}$

The mass of the electron is, $m_e = 9.1 \times 10^{-31}$ kg

The mass of the neutron is, $m_n = 1.66 \times 10^{-27}$ kg

The Plank's constant is, $h = 6.6 \times 10^{-34}$ Js

(a) The kinetic energy of the electron is, $K = \frac{1}{2} m_e v^2$

De Broglie wavelength of electron is, $\lambda = \frac{h}{m_e v}$

Thus, the velocity of electron is, $v = \frac{h}{m_e \lambda}$

Thus, the kinetic energy of the electron is,

$$K = \frac{1}{2} m_e \left(\frac{h}{m_e \lambda}\right)^2 = \frac{h^2}{2\lambda^2 m_e}$$

$$= \frac{(6.6 \times 10^{-34}\ \mathrm{Js})^2}{2(589 \times 10^{-9}\ \mathrm{m})(9.1 \times 10^{-31}\ \mathrm{kg})}$$

$$= 6.9 \times 10^{-25}\ \mathrm{J}$$

$$= (6.9 \times 10^{-25}\ \mathrm{J})\left(\frac{1\ \mathrm{eV}}{1.6 \times 10^{-19}\ \mathrm{J}}\right)$$

$= 4.31 \times 10^{-6}$ eV

$= (4.31 \times 10^{-6}\ \text{eV})\left(\dfrac{1\ \mu\text{eV}}{10^{-6}\ \text{eV}}\right)$

$= 4.31\ \mu\text{eV}$

The kinetic energy of electron is $4.31\ \mu$eV.

(b) The kinetic energy of neutron is,

$$K = \frac{h^2}{2\lambda^2 m_n}$$

$$= \frac{(6.6 \times 10^{-34}\ \text{Js})^2}{2(589 \times 10^{-9}\ \text{m})(1.66 \times 10^{-27}\ \text{kg})}$$

$$= 3.78 \times 10^{-28}\ \text{J}$$

$$= (3.78 \times 10^{-28}\ \text{J})\left(\frac{1\ \text{eV}}{1.6 \times 10^{-19}\ \text{J}}\right)$$

$$= 2.36 \times 10^{-9}\ \text{eV}$$

$$= (2.36 \times 10^{-9}\ \text{eV})\left(\frac{1\ \text{neV}}{10^{-9}\ \text{eV}}\right)$$

$$= 2.36\ \text{neV}$$

The kinetic energy of neutron is 2.36 neV.

15. What is the de Broglie wavelength of

(a) a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s,

(b) a ball of mass 0.060 kg moving at a speed of 1.0 m/s, and

(c) a dust particle of mass 1.0×10^{-9} kg drifting with a speed of 2.2 m/s?

Solution:

(a) The mass of bullet is, $m = 0.040$ kg

The speed of bullet is,

$v = 1$ km

$= (1\ \text{km})\left(\dfrac{1000\ \text{m}}{1\ \text{km}}\right)$

$= 1000$ m

The Plank's constant is, $h = 6.6 \times 10^{-34}$ Js

The de Broglie wavelength of the bullet is,

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6 \times 10^{-34}\ \text{J}}{(0.040\ \text{kg})(1000\ \text{m/s})}$$

$$= 1.65 \times 10^{-35}\ \text{m}$$

(b) The mass of ball is, $m = 0.060$ kg

The speed of ball is, $v = 1.0\ \text{m/s}$

De Broglie wavelength of the ball is,

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6 \times 10^{-34}\ \text{J}}{(0.060\ \text{kg})(1\ \text{m/s})}$$

$$= 1.1 \times 10^{-32}\ \text{m}$$

(c) The mass of dust particle is, $m = 1 \times 10^{-9}$ kg

The speed of dust particle is, $v = 2.2\ \text{m/s}$

The de Broglie wavelength of the dust particle is,

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6 \times 10^{-34}\ \text{J}}{(1 \times 10^{-9}\ \text{kg})(2.2\ \text{m/s})}$$

$$= 3.0 \times 10^{-25}\ \text{m}$$

16. An electron and a photon each have a wavelength of 1.00 nm. Find

(a) their momenta,

(b) the energy of the photon, and

(c) the kinetic energy of electron.

Solution:

The wavelength of the photon is $\lambda_p = 1.00\ \text{nm} = 1 \times 10^{-9}$ m.

The wavelength of the electron is $\lambda_e = 1.00\ \text{nm} = 1 \times 10^{-9}$ m.

(a) According to De Broglie relation, the momentum is

$$p = \frac{h}{\lambda}$$

As the momentum depends only on the wavelength, both photon and electron have the same momentum.

$$p = \frac{6.626 \times 10^{-34}}{1 \times 10^{-9}}$$

$$= 6.626 \times 10^{-25} \text{ kgm/s}$$

The momentum of photon and electron is 6.626×10^{-25} kgm/s.

(b) The energy of photon is given by,

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{1 \times 10^{-9} \times 1.6 \times 10^{-19}}$$

$$= 1243.1 \text{ eV}$$

$$= 1.243 \text{ keV}$$

(c) The kinetic energy of the electron is,

$$K = \frac{1}{2}\frac{p^2}{m}$$

$$= \frac{1}{2}\frac{(6.63 \times 10^{-25})^2}{9.1 \times 10^{-31} \times 1.6 \times 10^{-19}}$$

$$= \frac{2.415 \times 10^{-19} \text{ J}}{1.6 \times 10^{-19}}$$

$$= 1.51 \text{ eV}$$

17. (a) For what kinetic energy of a neutron will the associated de Broglie wavelength be 1.40×10^{-10} m?

(b) Also find the de Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of $(3/2)$ k T at 300 K.

Solution:

(a) De Broglie wavelength of the neutron is 1.40×10^{-10} m.

Neutron has a mass of $m_n = 1.66 \times 10^{-27}$ kg.

The relation between kinetic energy and velocity is,

$$K = \frac{1}{2} m_n v^2 \qquad \ldots\ldots \text{(i)}$$

The relation between De Broglie wavelength and velocity is,

$$\lambda = \frac{h}{m_n v} \qquad \ldots\ldots \text{(ii)}$$

Comparing equations (i) and (ii),

$$K = \frac{1}{2}\frac{m_n h^2}{\lambda^2 m_n^2} = \frac{h^2}{2\lambda^2 m_n}$$

Substituting the values,

$$K = \frac{(6.626 \times 10^{-34})^2}{2(1.40 \times 10^{-10})^2 \times 1.66 \times 10^{-27}}$$

$$= 6.75 \times 10^{-21}\ \text{J}$$

$$= \frac{6.75 \times 10^{-21}}{1.6 \times 10^{-19}}$$

$$= 4.219 \times 10^{-2}\ \text{eV}$$

(b) Average kinetic energy of the neutron is,

$$K' = \frac{3}{2}kT$$

$$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 300$$

$$= 6.21 \times 10^{-21}\ \text{J}$$

The De Broglie wavelength is,

$$\lambda = \frac{h}{\sqrt{2K'm_n}}$$

$$= \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 6.21 \times 10^{-21} \times 1.66 \times 10^{-27}}}$$

$$= 1.46 \times 10^{-10}\ \text{m}$$

$$= 0.146\ \text{mm}$$

18. Show that the wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum (photon).

Solution:

The momentum of the photon is,

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

On rearranging, $\lambda = \frac{h}{p}$ …… (i)

The De Broglie wavelength is, $\lambda = \frac{h}{mv}$

Substitute $p$for mv.

$\lambda = \frac{h}{p}$ ...... (ii)

As equation (i) and (ii) are the same, e wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum.

19. What is the de Broglie wavelength of a nitrogen molecule in air at 300 K? Assume that the molecule is moving with the root-mean square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u)

Solution:

Given

The atomic mass of nitrogen is 14.0076 u.

Thus, the mass of nitrogen molecule is,

$m = 2 \times 14.0076\text{ u}$

$= 28.0152\text{ u}$

$= 28.0152 \times 1.66 \times 10^{-27}$

The relation between the kinetic energy of the nitrogen molecule and the root mean square speed is,

$$\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$$

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

Thus, the De Broglie wavelength of the nitrogen molecule is,

$$\lambda = \frac{h}{mv_{rms}} = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.626 \times 10^{-34}}{\sqrt{3 \times 28.0152 \times 1.66 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}}$$

$= 0.028 \times 10^{-9}\text{ m}$

$= 0.028\text{ m}$

Thus, the De Broglie wavelength of the nitrogen molecule is, 0.028 m.

20. (a) Estimate the speed with which electrons emitted from a heated emitter of an evacuated tube impinge on the collector maintained at a potential difference of 500 V with respect to the emitter. Ignore the small initial speeds of the electrons. The *specific charge* of the electron, i.e., its e/m is given to be $1.76 \times 10^{11}\text{ C kg}^{-1}$.

(b) Use the same formula you employ in (a) to obtain electron speed for a collector potential of 10 MV. Do you see what is wrong ? In what way is the formula to be modified?

Solution:

(a) Given

The potential difference across the tube is, $V = 500$ V.

The specific charge of the electron is, $e/m = 1.76 \times 10^{11}$ C/kg.

The speed of the emitted electron is,

$$\mathrm{K} = \frac{1}{2}mv^2 = eV$$

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{2V\frac{e}{m}}$$

Substituting the values,

$$v = \sqrt{2 \times 500 \times 1.76 \times 10^{11}}$$

$$= 1.327 \times 10^7 \text{ m/s}$$

Thus, the speed of the emitted electron is 1.327×10^7 m/s.

(b) Given

The potential of the anode is, $V = 10$ MV $= 10 \times 10^6$ V.

The speed of each electron is, $v = \sqrt{2V\frac{e}{m}}$.

Substituting the values,

$$v = \sqrt{2 \times 10^7 \times 1.76 \times 10^{11}}$$

$$= 1.88 \times 10^9 \text{ m/s}$$

This speed is greater than the speed of light which is not possible. This is because the expression for energy $\frac{mv^2}{2}$can be used only in non-relativistic situations ($v << c$). In relativistic situations, the total energy is written as

$$E = mc^2$$

Here, $m$is the relativistic mass given by $m = m_0\sqrt{1 - \frac{v^2}{c^2}}$.

Here, $m_0$is the mass of the particle at rest.

Thus, the kinetic energy can be written as

$$K = mc^2 - m_0c^2$$

21. (a) A monoenergetic electron beam with electron speed of 5.20×10^6 m s^{-1} is subject to a magnetic field of 1.30×10^{-4} T normal to the beam velocity. What is the radius of the circle traced by the beam, given *e/m* for electron equals 1.76×10^{11}C kg^{-1}.

(b) Is the formula you employ in (a) valid for calculating radius of the path of

Solution:

(a) The speed of the electron is, $v = 5.20 \times 10^6$ m/s

The magnetic field is, $B = 1.30 \times 10^{-4}$ T

The specific charge of the electron is,

$$\frac{e}{m} = \frac{1.6 \times 10^{-19}\ \text{C}}{9.1 \times 10^{-31}\ \text{kg}}$$

$$= 1.76 \times 10^{11}\ \text{C/kg}$$

The force exerted on the electron is,

$$F = e\left|\vec{v} \times \vec{B}\right|$$

$$= evB\sin\theta$$

Here, θ is the angle between velocity and magnetic field. Since the magnetic field is normal to the direction of beam, the force is,

$$F = evB\sin 90° = evB$$

The magnetic force acting on the electron will allow it to move in a circular path. Centripetal force acting on the electron is, $F = \frac{mv^2}{r}$

The magnetic force is equal in magnitude with the centripetal force. Therefore,

$$evB = \frac{mv^2}{r}$$

Here, r is the radius of circular path of the electron. Thus, the radius of electron is,

$$r = \frac{mv}{eB} = \frac{v}{(e/m)B}$$

$$= \frac{5.20 \times 10^6\ \text{m/s}}{(1.76 \times 10^{11}\ \text{C/kg})(1.30 \times 10^{-4}\ \text{T})}$$

$= 0.227$ m or 22.7 cm

Thus, 22.7 cm is the radius of the circular path.

(b) The energy of the electron beam is,

$E = 20\text{MeV}$

$= (20\text{MeV})\left(\frac{1 \text{ eV}}{10^{-6} \text{ MeV}}\right)$

$= 20 \times 10^6$ eV

$= (20 \times 10^6 \text{ eV})\left(\frac{1.6 \times 10^{-19} \text{ J}}{1 \text{ eV}}\right)$

$= 32 \times 10^{-13}$ J

Kinetic energy of the electron is,

$E = \frac{1}{2}mv^2$

Therefore, the speed of the electron is,

$v = \sqrt{\frac{2E}{m}}$

$= \sqrt{\frac{2 \times 36 \times 10^{-13} \text{ J}}{9.1 \times 10^{-31} \text{ kg}}}$

$= 2.65 \times 10^9$ m/s

No objects can move faster than light. Therefore, the result is incorrect. Therefore, the energy equation can only use in non-relativistic case where $v << c$. In relativistic case, mass is given by,

$m = m_0\left(1 - \frac{v^2}{c^2}\right)^{1/2}$

Here, m_0 is the rest mass of particle.

Therefore, radius of the circular path is,

$r = \frac{mv}{eB} = \frac{m_0 v}{eB\sqrt{\frac{c^2 - v^2}{c^2}}}$

22. An electron gun with its collector at a potential of 100 V fires out electrons in a spherical bulb containing hydrogen gas at low pressure ($\sim 10^{-2}$ mm of Hg). A magnetic field of 2.83×10^{-4} T curves the path of the electrons in a circular orbit of radius 12.0 cm. (The path can be viewed because the gas ions in the path focus the beam by attracting electrons and emitting light by electron capture; this method is known as the 'fine beam tube' method.) Determine *e/m* from the data.

Solution:

The potential of the anode is, $V = 100$ V

The magnetic field is, $B = 2.83 \times 10^{-4}$T

The radius of the circular orbit is,

$r = 12.0$ cm

$= (12.0\text{ cm})\left(\frac{1\text{ m}}{100\text{ cm}}\right)$

$= 12.0 \times 10^{-2}$ m

The kinetic energy of the electron and the electron's energy is equal in magnitude. Therefore,

$$\frac{1}{2}mv^2 = eV$$

Here, m is the mass of electron, e is the charge of electron and v is the

Therefore,

$$v^2 = \frac{2eV}{m}$$

The centripetal force and the magnetic force acting on the electron is equal in magnitude. Therefore,

$$\frac{mv^2}{r} = evB$$

$$\Rightarrow eB = \frac{mv}{r}$$

$$\Rightarrow v = \frac{eBr}{m}$$

Therefore,

$$\frac{2eV}{m} = \left(\frac{eBr}{m}\right)^2$$

Therefore,

$$\frac{e}{m} = \frac{2V}{B^2r^2}$$

$$= \frac{2 \times (100\ \text{V})}{(2.83 \times 10^{-4}\ \text{T})^2 \times (12.0 \times 10^{-2}\ \text{m})^2}$$

$= 1.73 \times 10^{11}\ \text{C/kg}$

Thus, $1.73 \times 10^{11}\ \text{C/kg}$ is the specific charge ratio.

23. (a) An X-ray tube produces a continuous spectrum of radiation with its short wavelength end at 0.45 Å. What is the maximum energy of a photon in the radiation?

(b) From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube?

Solution:

(a) Given,

The wavelength of the X-ray is, $\lambda = 0.45\ \text{Å} = 0.45 \times 10^{-10}\ \text{m}$.

The maximum energy of the photon in the radiation is,

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{0.45 \times 10^{-10} \times 1.6 \times 10^{19}}$$

$= 27.6 \times 10^3\ \text{eV}$

$= 27.6\ \text{keV}$

Thus, the maximum energy of the photon in the radiation is 27.6 keV.

(b) The photons have a maximum energy of 27.6 keV. The energy for producing X-rays by electrons is given by accelerating voltage. For the X-ray to have an energy of 27.6 keV, the electrons must have at least an energy of 27.6 keV. Thus, for producing X-rays, an accelerating voltage of the order of 30 keVis required.

24. In an accelerator experiment on high-energy collisions of electrons with positrons, a certain event is interpreted as annihilation of an electron-positron pair of total energy 10.2 BeV into two γ-rays of equal energy. What is the wavelength associated with each γ-ray? (1 BeV $= 10^9$ eV)

Solution:

Given,

The total energy of two γ-rays is

$E = 10.2\ \text{BeV}$

$= 10.2 \times 10^9$ eV

$= 10.2 \times 10^9 \times 1.6 \times 10^{-19}$ J

The energy of each γ-ray is,

$$E' = \frac{E}{2}$$

$$= \frac{10.2 \times 10^9 \times 1.6 \times 10^{-19} \text{ J}}{2}$$

$= 8.16 \times 10^{-10}$ J

The wavelength associated with γ-ray is,

$$E' = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E'}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{8.16 \times 10^{-10}}$$

$= 2.436 \times 10^{-16}$ m

Thus, the wavelength associated with γ-ray is 2.436×10^{-16} m.

25. Estimating the following two numbers should be interesting. The first number will tell you why radio engineers do not need to worry much about photons! The second number tells you why our eye can never 'count photons', even in barely detectable light.

(a) The number of photons emitted per second by a Medium wave transmitter of 10 kW power, emitting radio waves of wavelength 500 m.

(b) The number of photons entering the pupil of our eye per second corresponding to the minimum intensity of white light that we humans can perceive ($\sim$10– 10 W m^{-2}). Take the area of the pupil to be about 0.4 cm^2, and the average frequency of white light to be about 6×10^{14} Hz.

Solution:

(a) Given

The wave transmitter's power is $P = 10 \text{ kW} = 10^4 \text{ W} = 10^4 \text{ J/s}$.

Thus, the energy emitted per second by the transmitter is 10^4 J.

The radio wave has a wavelength of $\lambda = 500$ m.

The energy of the wave is

$E_1 = \frac{hc}{\lambda}$

$= \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{500}$

$= 3.96 \times 10^{-28}$ J

Assume the *n* is the number of photons that the transmitter emits.

Thus,

$nE_1 = E$

$n = \frac{E}{E_1}$

$= \frac{10^4}{3.96 \times 10^{-28}}$

$= 2.525 \times 10^{31}$

$\approx 3 \times 10^{31}$

Even though the number of photons emitted per second is large, the energy of the emitted photon is less.

(b) Given

The minimum intensity of light perceived by the human eye is, $I = 10^{-10}\ \mathrm{W/m^2}$.

The average frequency of the white light is, $\nu = 6 \times 10^{14}$ Hz.

The area of the pupil is, $A = 0.4\ \mathrm{cm^2} = 0.4 \times 10^{-4}\ \mathrm{m^2}$.

The energy emitted by the photon is,

$E = h\nu$

$= 6.626 \times 10^{-34} \times 6 \times 10^{14}$

$= 3.96 \times 10^{-19}$ J

Let n be a number that represents the number of photons falling per second per unit area of the pupil.

For *n* falling protons, the total energy per unit area per unit second is,

$E = n \times 3.96 \times 10^{-19}\ \mathrm{J\,s^{-1}m^{-2}}$

As intensity is the energy per unit area per second,

$E = I$

$n \times 3.96 \times 10^{-19} = 10^{-10}$

$n = \dfrac{10^{-10}}{3.96 \times 10^{-19}}$

$= 2.52 \times 10^{8}\ \text{m}^2\ \text{s}^{-1}$

Total number of photons entering the pupil per second is given by

$n_A = nA$

$= 2.52 \times 10^{8} \times 0.4 \times 10^{-4}$

$= 1.008 \times 10^{4}\ s^{-1}$

Even though the number is not as large as the number in (a), it is large enough that the human eye cannot see individual photons.

26. Ultraviolet light of wavelength 2271 Å from a 100 W mercury source irradiates a photo-cell made of molybdenum metal. If the stopping potential is – 1.3 V, estimate the work function of the metal. How would the photo-cell respond to a high intensity (~105 W m^{-2}) red light of wavelength 6328 Å produced by a He-Ne laser?

Solution:

Given

The wavelength of the ultraviolet light is, $\lambda = 2271\ \text{Å} = 2271 \times 10^{-10}$ m.

The metal has a stopping potential of $V_0 = 1.3$ V.

The equation for work function according to photoelectric effect is,

$\phi_0 = h\nu - eV_0$

$= \dfrac{hc}{\lambda} - eV_0$

$= \dfrac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{2271 \times 10^{-10}} - 1.6 \times 10^{-19} \times 1.3$

$= 8.72 \times 10^{-19} - 2.08 \times 10^{-19}$

$= 6.64 \times 10^{-19}$ J

$= \dfrac{6.64 \times 10^{-19}}{1.6 \times 10^{-19}}$

$= 4.15$ eV

If $\nu_0$is the threshold frequency of the metal, then

$\phi_0 = h\nu_0$

$$\Rightarrow \nu_0 = \frac{\phi_0}{h}$$

$$= \frac{6.64 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$$= 1.006 \times 10^{15}\ \text{Hz}$$

The frequency of the red light is, $\nu_r = \frac{c}{\lambda_r}$.

Substitute 6328×10^{-10} mfor $\lambda_r$in the above equation,

$$\nu_r = \frac{3 \times 10^8}{6328 \times 10^{-10}} = 4.74 \times 10^{14}\ \text{Hz}$$

Thus, the photocell will not respond to the red light from the laser as $\nu_0 > \nu_r$.

27. Monochromatic radiation of wavelength 640.2 nm ($1\ \text{nm} = 10^{-9}$ m) from a neon lamp irradiates photosensitive material made of cesium on tungsten. The stopping voltage is measured to be 0.54 V. The source is replaced by an iron source and its 427.2 nm line irradiates the same photocell. Predict the new stopping voltage.

Solution:

Given

Wavelength of the monochromatic radiation is, $\lambda = 640.2\ \text{nm} = 640.2 \times 10^{-9}$ m.

The stopping voltage for the lamp is, $V_0 = 0.54$ V.

According to photoelectric effect, the relation between work function and frequency is,

$$eV_0 = h\nu - \phi_0$$

$$\phi_0 = \frac{hc}{\lambda} - eV_0$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{640.2 \times 10^{-9}} - 1.6 \times 10^{-19} \times 0.54$$

$$= 3.093 \times 10^{-19} - 0.864 \times 10^{-19}$$

$$= 2.229 \times 10^{-19}\ \text{J}$$

$$= \frac{2.229 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$= 1.39\ \text{eV}$$

The wavelength of the wave from the iron source is$\lambda' = 427.2\ \text{nm} = 427.2 \times 10^{-9}$ m.

From the photoelectric equation, the new stopping potential is,

$$eV_0{}' = \frac{hc}{\lambda} - \phi_0$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{427.2 \times 10^{-9}} - 2.229 \times 10^{-19}$$

$$= 4.63 \times 10^{-19} - 2.229 \times 10^{-19}$$

$$= 2.401 \times 10^{-19}\ \text{J}$$

$$= \frac{2.401 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$= 1.5\ \text{eV}$$

Thus, the new stopping potential is 1.5 eV.

28. A mercury lamp is a convenient source for studying frequency dependence of photoelectric emission, since it gives a number of spectral lines ranging from the UV to the red end of the visible spectrum. In our experiment with rubidium photo-cell, the following lines from a mercury source were used: $\lambda_1 = 3650$ Å, $\lambda_2 = 4047$ Å, $\lambda_3 = 4358$ Å, $\lambda_4 = 5461$ Å, $\lambda_5 = 6907$ Å, The stopping voltages, respectively, were measured to be: $V_{01} = 1.28\,V$, $V_{02} = 0.95\,V$, $V_{03} = 0.74\,V$, $V_{04} = 0.16\,V$, $V_{05} = 0\,V$. Determine the value of Planck's constant h, the threshold frequency and work function for the material.

Solution:

According to Einstein's photoelectric equation,

$$eV_0 = h\nu - \phi_0$$

$$\Rightarrow V_0 = \frac{h}{e}\nu - \frac{\phi_0}{e}$$

Here, V_0 is the stopping potential, h is the Plank's constant, e is the charge of electron, ν is the frequency of the radiation and ϕ_0 is the work function of a material.

The stopping potential is directionally proportional to frequency.

The frequency is, $\nu = \frac{c}{\lambda}$

Here, c is the speed of light and λ is the wavelength.

Therefore, the frequencies for various lines of given wavelengths can be obtained.

$$\nu_1 = \frac{c}{\lambda_1} = \frac{3 \times 10^{8}\ \text{m/s}}{3650 \times 10^{-10}\ \text{m}} = 8.219 \times 10^{14}\ \text{Hz}$$

$$\nu_2 = \frac{c}{\lambda_2} = \frac{3 \times 10^{8}\ \text{m/s}}{4047 \times 10^{-10}\ \text{m}} = 7.412 \times 10^{14}\ \text{Hz}$$

$$\nu_3 = \frac{c}{\lambda_3} = \frac{3 \times 10^8 \text{ m/s}}{4358 \times 10^{-10} \text{ m}} = 6.884 \times 10^{14} \text{ Hz}$$

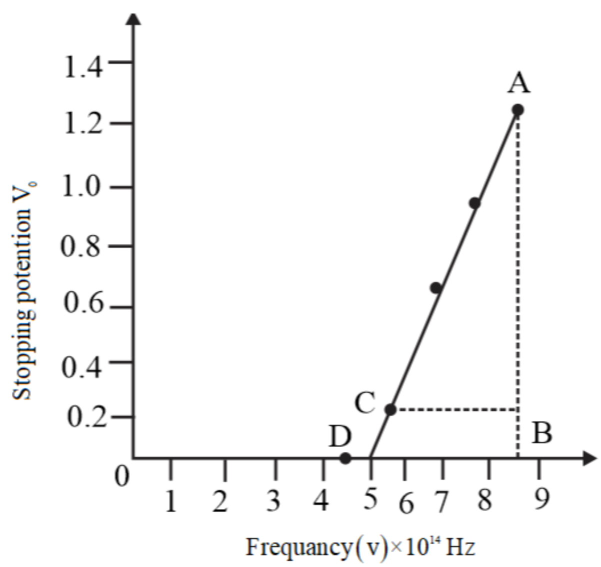
$$\nu_4 = \frac{c}{\lambda_4} = \frac{3 \times 10^8 \text{ m/s}}{5461 \times 10^{-10} \text{ m}} = 5.493 \times 10^{14} \text{ Hz}$$

$$\nu_5 = \frac{c}{\lambda_5} = \frac{3 \times 10^8 \text{ m/s}}{6907 \times 10^{-10} \text{ m}} = 4.343 \times 10^{14} \text{ Hz}$$

The tabular form of the data is,

Frequency $\times 10^{14}$ Hz	8.219	7.412	6.884	5.493	4.343
StoppingPotentialV_0	1.28	0.95	0.74	0.16	0

The graph between frequency and stopping potential is,



The curve of the graph is straight line and it intersects the axis of frequency at 5×10^{14} Hz. Therefore, 5×10^{14} Hz is the threshold frequency of the material. Therefore, for λ_5 there will not be any photo electric emission and no stopping voltage is required.

Slope of the line is,

$$\frac{AB}{CB} = \frac{1.28 - 0.16 \text{ V}}{(8.214 - 5.493) \times 10^{14} \text{ Hz}}$$

$$= 4.12 \times 10^{-15} \text{ V/Hz}$$

From equation for stopping potential, the slope is,

$$\frac{h}{e} = 4.12 \times 10^{15} \text{ V/Hz}$$

Therefore, the Plank's constant is,

$h = (4.12 \times 10^{15}\ \text{V/Hz})e$

$= (4.12 \times 10^{15}\ \text{V/Hz})(1.6 \times 10^{-19}\ \text{C})$

$= 6.573 \times 10^{-34}\ \text{Js}$

The work function of the metal is,

$\phi_0 = h\nu_0$

$= (6.573 \times 10^{-34}\ \text{Js})(5 \times 10^{14}\ \text{Hz})$

$= 3.286 \times 10^{-19}\ \text{J}$

$= (3.286 \times 10^{-19}\ \text{J})\left(\frac{1\ \text{eV}}{1.6 \times 10^{-19}\ \text{J}}\right)$

$= 2.054\ \text{eV}$

29. The work function for the following metals is given: Na: 2.75 eV; K: 2.30 eV; Mo: 4.17 eV; Ni: 5.15 eV. Which of these metals will not give photoelectric emission for a radiation of wavelength 3300 Å from a He-Cd laser placed 1 m away from the photocell? What happens if the laser is brought nearer and place 50 cm away?

Solution:

The wavelength of the radiation is,

$\lambda = 3300\ \text{Å}$

$= (3300\ \text{Å})\left(\frac{1\ \text{m}}{10^{10}\ \text{Å}}\right)$

$= 3300 \times 10^{-10}\ \text{m}$

The speed of light is, $c = 3 \times 10^{8}\ \text{m/s}$

The Plank's constant is, $h = 6.6 \times 10^{-34}\ \text{Js}$

The energy of the incident radiation is,

$E = \frac{hc}{\lambda}$

$= \frac{6.6 \times 10^{-34}\ \text{J} \times 3 \times 10^{8}\ \text{m/s}}{3300 \times 10^{-10}\ \text{m}}$

$= 6 \times 10^{-19}\ \text{J}$

$= (6 \times 10^{-19}\ \text{J})\left(\frac{1\ \text{eV}}{1.6 \times 10^{-19}\ \text{J}}\right)$

$= 3.16\ \text{eV}$

The energy exceeds the work function of K and Na alone. Therefore, both Ni and Mo will not show photo electric emission. The intensity of incident light increases as the source brought near. However, it will not affect the energy of the radiation, but the photoelectron emitted from K and Na increase in proportion to intensity.

30. Light of intensity 10^{-5} W m^{-2} falls on a sodium photo-cell of surface area 2 cm^2. Assuming that the top 5 layers of sodium absorb the incident energy, estimate time required for photoelectric emission in the wave-picture of radiation. The work function for the metal is given to be about 2 eV. What is the implication of your answer?

Solution:

The intensity of the incident light is, $I = 10^{-5}\ \mathrm{W/m^2}$

The surface area of the sodium cell is,

$A = 2\ \mathrm{cm^2}$

$= (2\ \mathrm{cm^2})\left(\dfrac{1\ \mathrm{m^2}}{10^4\ \mathrm{cm^2}}\right)$

$= 2 \times 10^{-4}\ \mathrm{m^2}$

The power if the incident life is,

$P = IA$

$= 10^{-5}\ \mathrm{W/m^2} \times 2 \times 10^{-4}\ \mathrm{m^2}$

$= 2 \times 10^{-9}\ \mathrm{W}$

The work function of the metal,

$\phi_0 = 2\ \mathrm{eV}$

$= (2\ \mathrm{eV})\left(\dfrac{1.6 \times 10^{-19}\ \mathrm{J}}{1\ \mathrm{eV}}\right)$

$= 3.2 \times 10^{-19}\ \mathrm{J}$

The number of layers of sodium that absorbs the incident energy, $n = 5$

The effective atomic area of sodium atom is, $A_e = 10^{-20}\ \mathrm{m^2}$

Therefore, the number of conduction electrons in n layers is,

$n' = n \times \dfrac{A}{A_e}$

$= 5 \times \dfrac{2 \times 10^{-4}\ \mathrm{m^2}}{10^{-20}\ \mathrm{m^2}}$

$= 10^{17}$

The electrons uniformly absorb incident power. Amount of energy absorbed per second per electron is given by,

$$E = \frac{P}{n'}$$

$$= \frac{2 \times 10^{-9}\ \text{W}}{10^{17}}$$

$$= 2 \times 10^{-26}\ \text{J/s}$$

The time required for photoelectric emission is,

$$t = \frac{\phi_0}{E}$$

$$= \frac{3.2 \times 10^{-19}}{2 \times 10^{-26}}$$

$$= 1.6 \times 10^{7}\ \text{s}$$

$$= (1.6 \times 10^{7}\ \text{s})\left(\frac{1\ \text{year}}{3.15 \times 10^{7}\ \text{s}}\right)$$

$$= 0.51\ \text{years}$$

The photoelectric emission will not take half a year. Therefore, the wave picture is not incorrect.

31. Crystal diffraction experiments can be performed using X-rays, or electrons accelerated through appropriate voltage. Which probe has greater energy? (For quantitative comparison, take the wavelength of the probe equal to 1 Å, which is of the order of inter-atomic spacing in the lattice) (*me* = 9.11 × 10^{-31} kg).

Solution:

Given

The wavelength of the probe is, $\lambda = 1\ \text{Å} = 1 \times 10^{-10}$ m.

The mass of the electron is, $m_e = 9.1 \times 10^{-31}$ kg.

For the same wavelength, an X-ray probe has greater energy than an electron probe.

The kinetic energy of the electron is,

$$E = \frac{1}{2} m_e v^2$$

$$\Rightarrow v = \sqrt{\frac{2E}{m_e}}$$

$$m_e v = \sqrt{2E m_e}$$

Here, $m_e v$ is the momentum of the electron.

The De Broglie wavelength, according to De Broglie relation is,

$$\lambda = \frac{h}{p} = \frac{h}{m_e v} = \frac{h}{\sqrt{2Em_e}}$$

$$E = \frac{h^2}{2\lambda^2 m_e}$$

$$= \frac{(6.626 \times 10^{-34})^2}{2(1 \times 10^{-10})^2 \times 9.11 \times 10^{-31}}$$

$$= 2.39 \times 10^{-17}\ \text{J}$$

$$= \frac{2.39 \times 10^{-17}}{1.6 \times 10^{-19}}$$

$$= 12.375 \times 10^3 \text{eV}$$

$$= 12.375\ \text{keV}$$

Thus, it can be concluded that, for the same wavelength, a proton has greater than an electron.

32. (a) Obtain the de Broglie wavelength of a neutron of kinetic energy 150 eV. As you have seen in Exercise 11.31, an electron beam of this energy is suitable for crystal diffraction experiments. Would a neutron beam of the same energy be equally suitable? Explain. ($m_n = 1.675 \times 10^{-27}$ kg)

(b) Obtain the de Broglie wavelength associated with thermal neutrons at room temperature (27 °C). Hence explain why a fast neutron beam needs to be thermalised with the environment before it can be used for neutron diffraction experiments.

Solution:

(a) The de Broglie wavelength is, 2.327×10^{-12} m

The kinetic energy of neutron is,

$$K = 150\ \text{eV}$$

$$= (150\ \text{eV})\left(\frac{1.6 \times 10^{-19}\ \text{J}}{1\ \text{eV}}\right)$$

$$= 2.4 \times 10^{-17}\ \text{J}$$

The mass of the neutron is, $m_n = 1.675 \times 10^{-27}$ kg

The kinetic energy of the neutron is,

$$K = \frac{1}{2} m_n v^2$$

Here, v is the velocity of neutron.

Therefore, the momentum of the neutron is,

$$m_n v = \sqrt{2 K m_n}$$

De Broglie wavelength of the neutron is,

$$\lambda = \frac{h}{m_n v} = \frac{h}{\sqrt{2 K m_n}}$$

Therefore, the wavelength is inversely proportional to the square root of the mass.

The wavelength is,

$$\lambda = \frac{6.6 \times 10^{-34}\ \mathrm{J}}{\sqrt{2 \times 2.4 \times 10^{-17}\ \mathrm{J} \times 1.675 \times 10^{-27}\ \mathrm{kg}}}$$

$$= 2.327 \times 10^{-12}\ \mathrm{m}$$

The inter atomic spacing of a crystal is about 10^{-10} m. Thus, 150 eVneutron beam is not suitable for diffraction.

(b) The de Broglie wavelength is, 1.44×10^{-10} m

The Boltzmann constant is 1.38×10^{-23} J/molK

The room temperature is,

$$T = 27°\mathrm{C}$$

$$= (27 + 273)\ \mathrm{K}$$

$$= 300\ \mathrm{K}$$

The average kinetic energy of neutron is,

$$E = \frac{3}{2} kT$$

The wavelength of the neutron is,

$$\lambda = \frac{h}{\sqrt{2 m_n E}} = \frac{h}{\sqrt{3 m_n k T}}$$

$$= \frac{6.6 \times 10^{-34}\ \mathrm{Js}}{\sqrt{3(1.675 \times 10^{-27}\ \mathrm{kg})(1.38 \times 10^{-23}\ \mathrm{J/molK})(300\ \mathrm{K})}}$$

$$= 1.447 \times 10^{-10}\ \mathrm{m}$$

Therefore, the neutron beam can be used to diffraction after thermalizing.

33. An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de Broglie wavelength associated with the electrons. If other factors (such as numerical aperture, etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?

Solution:

Given

The voltage used to accelerate the electrons is, $V = 50 \text{ kV} = 50 \times 10^3 \text{ V}$.

The yellow light has a wavelength of, $\lambda = 5.9 \times 10^{-7} \text{ m}$.

The kinetic energy of the electron is,

$E = eV$

$= 1.6 \times 10^{-19} \times 50 \times 10^3$

$= 8 \times 10^{-15} \text{ J}$

According to De Broglie relation, the wavelength is,

$$\lambda = \frac{h}{\sqrt{2m_e E}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 8 \times 10^{-15}}}$$

$= 5.467 \times 10^{-12} \text{ m}$

The relation between wavelength and resolving power of a microscope is that the resolving power of a microscope depends inversely on the wavelength of the light used. The De Broglie wavelength is about 10^5times smaller than the wavelength of the yellow light. Thus, an electron microscope has 10^5times the resolving power compared to an optical microscope.

34. The wavelength of a probe is roughly a measure of the size of a structure that it can probe in some detail. The quark structure of protons and neutrons appears at the minute length-scale of 10^{-15} m or less. This structure was first probed in early 1970's using high energy electron beams produced by a linear accelerator at Stanford, USA. Guess what might have been the order of energy of these electron beams. (Rest mass energy of electron = 0.511 MeV.)

Solution:

The wavelength of the proton or a neutron, $\lambda \approx 10^{-15}$ m

The rest mass energy of an electron is,

$m_0c^2 = 0.511 \text{ MeV}$

$= (0.511 \text{ MeV})\left(\frac{10^6 \text{ eV}}{1 \text{ MeV}}\right)\left(\frac{1.6 \times 10^{-19} \text{ J}}{1 \text{ eV}}\right)$

$= 0.8176 \times 10^{-13} \text{ J}$

The Plank's constant is, $h = 6.6 \times 10^{-34}$ Js

The speed of light, $c = 3 \times 10^8 \text{ m/s}$

The momentum of a proton or a neutron is,

$p = \frac{h}{\lambda}$

$= \frac{6.6 \times 10^{-34} \text{ Js}}{10^{-15} \text{ m}}$

$= 6.6 \times 10^{-19} \text{ kg m/s}$

The relativistic equation for energy is,

$E^2 = p^2c^2 + m_0^2c^4$

$\Rightarrow E = \sqrt{p^2c^2 + m_0^2c^4}$

$= \sqrt{(6.6 \times 10^{-19} \text{ kgm/s} \times 3 \times 10^8 \text{ m/s})^2 + (0.8176 \times 10^{-13})^2}$

$= \sqrt{392 \times 10^{-22} \text{ J}^2}$

$= 1.98 \times 10^{-10} \text{ J}$

$= (1.98 \times 10^{-10} \text{ J})\left(\frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}}\right)$

$= 1.24 \times 10^9 \text{ eV}$

$= (1.24 \times 10^9 \text{ eV})\left(\frac{1 \text{ BeV}}{10^9 \text{ eV}}\right)$

$= 1.24 \text{ BeV}$

Therefore, 1.24 BeVelectron energy is emitted.

35. Find the typical de Broglie wavelength associated with a He atom in helium gas at room temperature (27 °C) and 1 atm pressure; and compare it with the mean separation between two atoms under these conditions.

Solution:

De Broglie wavelength associated with He atom is, 0.7268×10^{-10} m

The room temperature is,

$T = 27\ °\mathrm{C}$

$= (27 + 273)\ \mathrm{K}$

$= 300\ \mathrm{K}$

The atmospheric pressure is,

$P = 1\ \mathrm{atm}$

$$= (1\ \mathrm{atm})\left(\frac{1.01 \times 10^5\ \mathrm{Pa}}{1\ \mathrm{atm}}\right)$$

$= 1.01 \times 10^5\ \mathrm{Pa}$

The atomic weight of a He atom $W = 4\ \mathrm{g}$

The Avogadro's number is, $N_A = 6.023 \times 10^{23}$

The Boltzmann constant is, $k = 1.38 \times 10^{-23}\ \mathrm{J/molK}$

The average energy of a gas is,

$$E = \frac{3}{2}kT$$

Here, T is the temperature.

The de Broglie wavelength is,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

Here, $m$is the mass of He atom.

$$m = \frac{W}{N_A}$$

$$= \frac{4\ \mathrm{g}}{6.023 \times 10^{23}}$$

$= 6.64 \times 10^{-24}\ \mathrm{g}$

$$= (6.64 \times 10^{-24}\ \mathrm{g})\left(\frac{1\ \mathrm{kg}}{1000\ \mathrm{g}}\right)$$

$= 6.64 \times 10^{-27}\ \mathrm{g}$

The de Broglie wavelength is,

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.6 \times 10^{-34} \text{ Js}}{\sqrt{(3)(6.64 \times 10^{-27} \text{ kg})(1.38 \times 10^{-23} \text{ J/mol K}) \times 300 \text{ K}}}$$

$$= 0.7268 \times 10^{-10} \text{ m}$$

The ideal gas formula is,

$$PV = RT$$

$$PV = kNT$$

$$\frac{V}{N} = \frac{kT}{P}$$

Here, V is the volume of the gas and N is the number of moles of the gas.

The mean separation between two atoms of the gas is,

$$r = \left(\frac{V}{N}\right)^{1/3}$$

$$= \left(\frac{kT}{P}\right)^{1/3}$$

$$= \left(\frac{(1.38 \times 10^{-23} \text{ J/molK})(300 \text{ K})}{1.01 \times 10^{5} \text{ Pa}}\right)$$

$$= 3.35 \times 10^{-9} \text{ m}$$

Therefore, the mean separation between two atoms of the gas is greater than de Broglie wavelength.

36. Compute the typical de Broglie wavelength of an electron in a metal at 27 °C and compare it with the mean separation between two electrons in a metal which is given to be about 2×10^{-10} m.

Solution:

The Plank's constant is, $h = 6.6 \times 10^{-34}$ Js

The mass of the electron is, $m = 9.11 \times 10^{-31}$ kg

The Boltzmann constant is, $k = 1.38 \times 10^{-23}$ J/molK

The room temperature is,

$$T = 27°\text{C}$$

$$= (27 + 273) \text{ K}$$

$$= 300 \text{ K}$$

The mean separation between two electrons is, $r = 2 \times 10^{-10}$ m

The de Broglie wavelength is,

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.6 \times 10^{-34}\ \text{J s}}{\sqrt{2(9.11 \times 10^{-31}\ \text{kg})(1.38 \times 10^{-23}\ \text{J/molK})(300\ \text{K})}}$$

$$= 6.2 \times 10^{-9}\ \text{m}$$

Therefore, the inter-electron separation is much less than de Broglie wavelength.

37. Answer the following questions:

(a) Quarks inside protons and neutrons are thought to carry fractional charges $[(+2/3)e\ ;\ (-1/3)e]$. Why do they not show up in Millikan's oil-drop experiment?

(b) What is so special about the combination e/m? Why do we not simply talk of e and m separately?

(c) Why should gases be insulators at ordinary pressures and start conducting at very low pressures?

(d) Every metal has a definite work function. Why do all photoelectrons not come out with the same energy if incident radiation is monochromatic? Why is there an energy distribution of photoelectrons?

(e) The energy and momentum of an electron are related to the frequency and wavelength of the associated matter wave by the relations: $E = h\nu, p = \frac{h}{\lambda}$. But while the value of λ is physically significant, the value of ν (and therefore, the value of the phase speed $\nu\lambda$) has no physical significance. Why?

Solution:

(a) The quarks situated inside neutron and proton carries fractional charges since if the pulled apart, the nuclear force increase extremely. Therefore, fractional charges can be present in nature and the observable charges are the integral multiple of electric charge.

(b) The kinetic energy of an electron moving in a potential is,

$$eV = \frac{1}{2}mv^2$$

Here, e is the charge of electron, V is the potential, $m$is the mass of electron and v is the velocity of electron.

The velocity is,

$$v = \sqrt{2V\left(\frac{e}{m}\right)}$$

The centripetal force acting on an electron in a magnetic field is,

$$eBv = \frac{mv^2}{r}$$

Here, B is the magnetic field and r is the radius of rotation of the electron.

The velocity is,

$$v = Br\left(\frac{e}{m}\right)$$

Therefore, the dynamics of the electron is determined by the ratio e/m and not by e and m separately.

(c) Gases are insulator at atmospheric pressure since the ions of gases will not have any chance to reach their respective electrons at due to the collision. The ions have a chance to reach their respective electrons and produce an electric current only at low pressure.

(d) The work function of a metal is defined as the minimum energy required for an electron to get out of the surface of the metal. The electrons in an atom will have different energy levels. When a photon ray of some energy is incident on the metal, electrons of different energies are emitted from different levels. Thus, the photoelectrons show different energy distributions.

(e) As the absolute value of energy for a particle is arbitrary within the additive constant, the frequency associated with an electron has no physical significance whereas the wavelength is significant.

Thus, the product of frequency and wavelength $\nu\lambda$has no physical significance.

The group speed can be shown as

$$v_{\mathrm{G}} = \frac{d\nu}{dk}$$

$$= \frac{d\nu}{d\left(\frac{1}{\lambda}\right)} = \frac{dE}{dp}$$

$$= \frac{d\left(\frac{p^2}{2m}\right)}{dp} = \frac{p}{m}$$

This is a quantity with physical meaning.

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